The emergence of quantum technologies: challenges and opportunities

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Industrial revolutions

First industrial revolution (~ 1760): steam engine (James Watt)

Second industrial revolution (~ 1900): electricity (Thomas Edison)

Third industrial revolution (~ 1940)**:** information technology and computers (Steve Jobs & Bill Gates)

Fourth industrial revolution (~ 1980): digital revolution, Internet and fusion of different technologies (Google, Facebook, Uber, Airbnb, Amazon)

Investment in quantum technologies

Governments:

- The US quantum initiative program ($1b)
- The European quantum flagship program (€1b)
- Quantum hubs in the UK (£560m)
- The National Quantum Strategy plan in Australia ($1b)

Companies:

- Giant Companies: Google, IBM, Microsoft, Amazon, Huawei, Tencent, Alibaba
- Startups: IonQ, PsiQ, Xanada, Zapata, …

Moor’s law: Processing power

1970

50 years

2020

The CPU of iphone is ~3000 times faster than Apollo spacecraft’s

Moor’s law: Memory

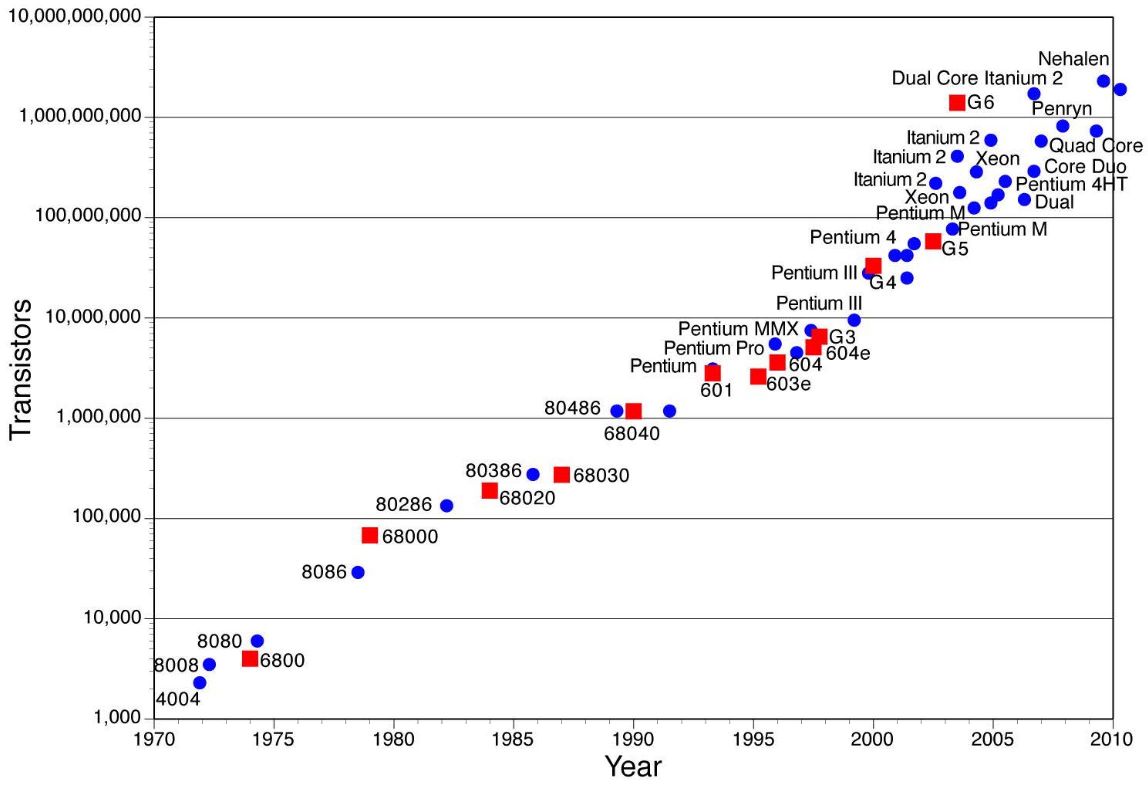
20 years

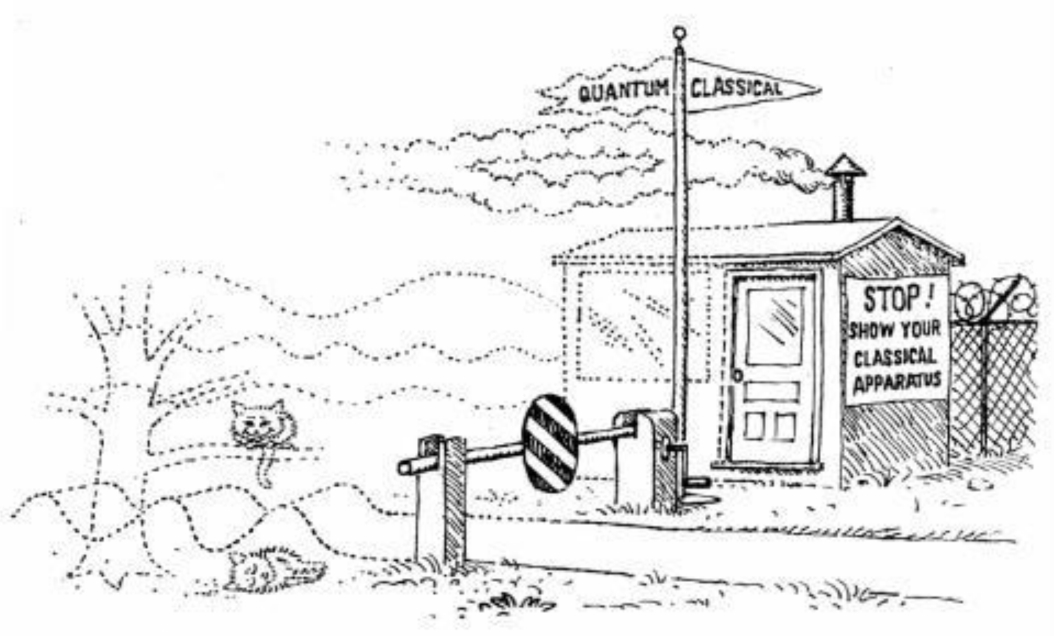
Launched 1999
Weight~ 150 g
Memory= 16 MB

Launched 2020
Weight~ 160 g
Memory= 256 GB

16000 times more memory with the same weight !!

Moor’s law





Delineating the border between the quantum realm ruled by the Schrödinger equation and the classical realm ruled by Newton's laws is one of the unresolved problems of physics. **Figure 1**

Zurek, Physics Today (1991)

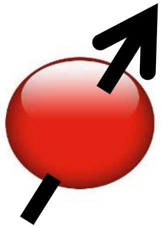
The number of transistors were doubled every two years for a period of ~50 years

Moor’s law is now violated as transistors have reached the atomic scale

Quantum Physics

Quantum Superposition

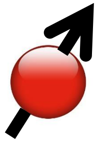
0 or 1



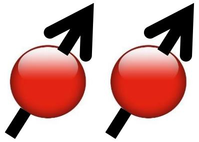
$\alpha|0\rangle + \beta|1\rangle$

0 & 1

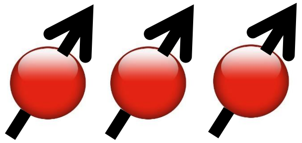
$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$, $|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$



$a_0|0\rangle + a_1|1\rangle$ → 2 numbers are encoded by 1 particle



$a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$ → 2^2 numbers are encoded by 2 particle



$a_{000}|000\rangle + a_{001}|001\rangle + a_{010}|010\rangle + a_{011}|011\rangle +$
$a_{100}|100\rangle + a_{101}|101\rangle + a_{110}|110\rangle + a_{111}|111\rangle$ → 2^3 numbers are encoded by 3 particle

By N particles one can encode 2^N numbers → $2^{300} \sim$ is equal to the number of atoms In the whole universe

Can quantum mechanics help for big data analysis?

Entanglement

The first consequence of super position principle is quantum entanglement

Can any bipartite system AB be described as: $|\Psi_{AB}\rangle \overset{?}{=} |\phi_A\rangle\,|\phi_B\rangle$

$$\left.\begin{aligned} |\Psi_{AB}\rangle &= (|00\rangle + |11\rangle)/\sqrt{2} \\ |\phi_A\rangle &= a_0|0\rangle + a_1|1\rangle \\ |\phi_B\rangle &= b_0|0\rangle + b_1|1\rangle \end{aligned}\right\} \Rightarrow |\phi_A\rangle\,|\phi_B\rangle = a_0 b_0|00\rangle + a_0 b_1|01\rangle + a_1 b_0|10\rangle + a_1 b_1|11\rangle = |\Psi_{AB}\rangle$$

$$\Rightarrow \left\{\begin{aligned} a_0 b_0 &= \frac{1}{\sqrt{2}} \\ a_0 b_1 &= 0 \\ a_1 b_0 &= 0 \\ a_1 b_1 &= 1/\sqrt{2} \end{aligned}\right. \Rightarrow$$

It is impossible to be satisfied

While we can precisely describe the whole system, we cannot describe the subsystems by a single quantum state

Quantum evolution (closed systems)

Every quantum state can evolve to another state through unitary operation

$$|\Psi\rangle = U\,|\phi\rangle \qquad UU^{\dagger} = U^{\dagger}U = I$$

For instance (mathematical description):

$$|\Psi(\theta)\rangle = e^{-i\theta\sigma_x}\,|0\rangle = \cos(\theta)|0\rangle - i\,\sin(\theta)|1\rangle \qquad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

In the lab, this unitary rotation is implemented by a magnetic field along the x direction

Quantum measurement

Unlike classical physics, quantum measurement changes the state of the system

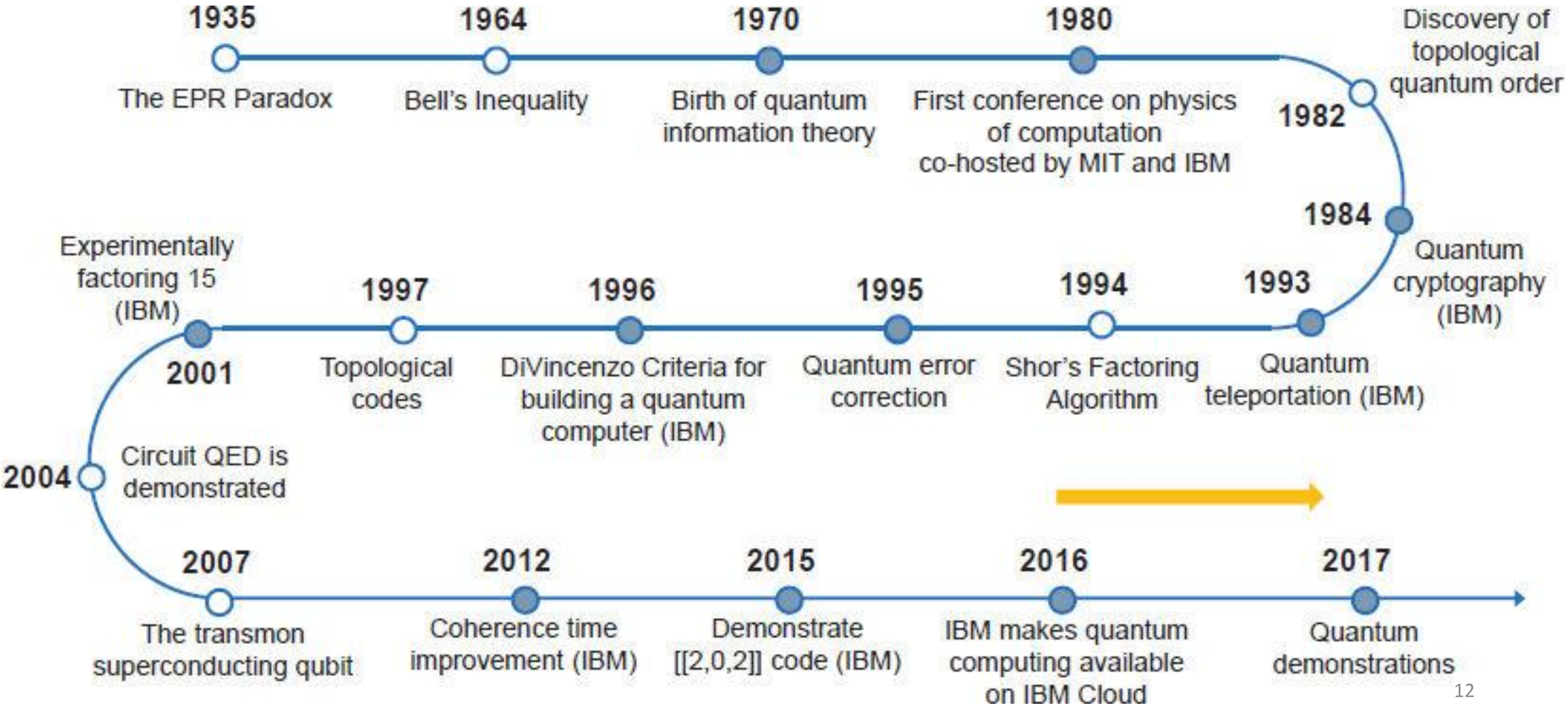
$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle \xrightarrow{\sigma_z \text{ measurement}} \begin{cases} p_0 = |a_0|^2: \quad |0\rangle \\ p_1 = |a_1|^2: \quad |1\rangle \end{cases}$$

Basis of the measurement matter:

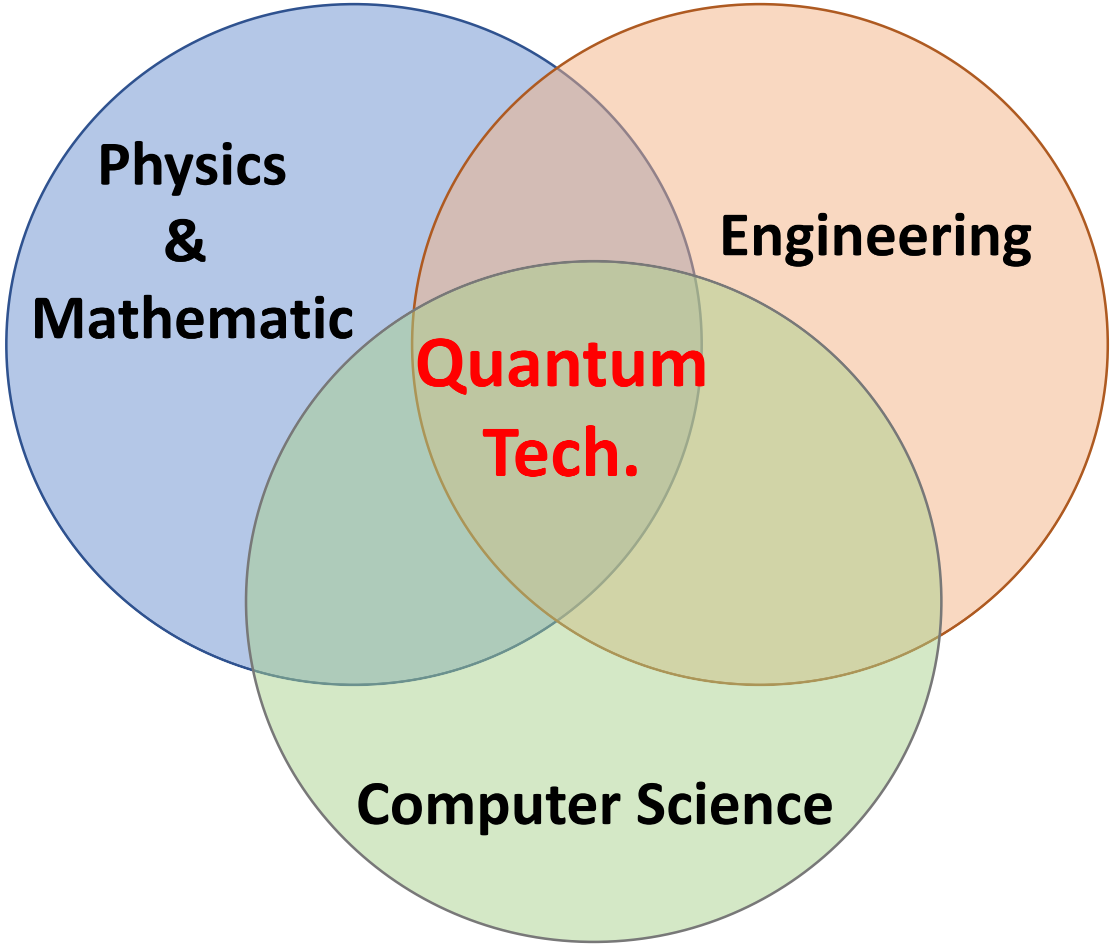
$$\left.\begin{aligned} |+\rangle &= \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\ |-\rangle &= \frac{|0\rangle - |1\rangle}{\sqrt{2}} \end{aligned}\right\} \Rightarrow \begin{aligned} |0\rangle &= \frac{|+\rangle + |-\rangle}{\sqrt{2}} \\ |1\rangle &= \frac{|+\rangle - |-\rangle}{\sqrt{2}} \end{aligned} \Rightarrow |\psi\rangle = a_0|0\rangle + a_1|1\rangle = \frac{a_0 + a_1}{\sqrt{2}}|+\rangle + \frac{a_0 - a_1}{\sqrt{2}}|-\rangle$$

$$\xrightarrow{\sigma_x \text{ measurement}} \begin{cases} p_+ = \dfrac{|a_0 + a_1|^2}{2}: \quad |+\rangle \\ p_- = \dfrac{|a_0 - a_1|^2}{2}: \quad |-\rangle \end{cases}$$

Quantum technologies timeline



Interdisciplinary subject



What is quantum technology?

Quantum communications:

- Quantum key distributions
- Quantum Internet

Quantum computation/simulations:

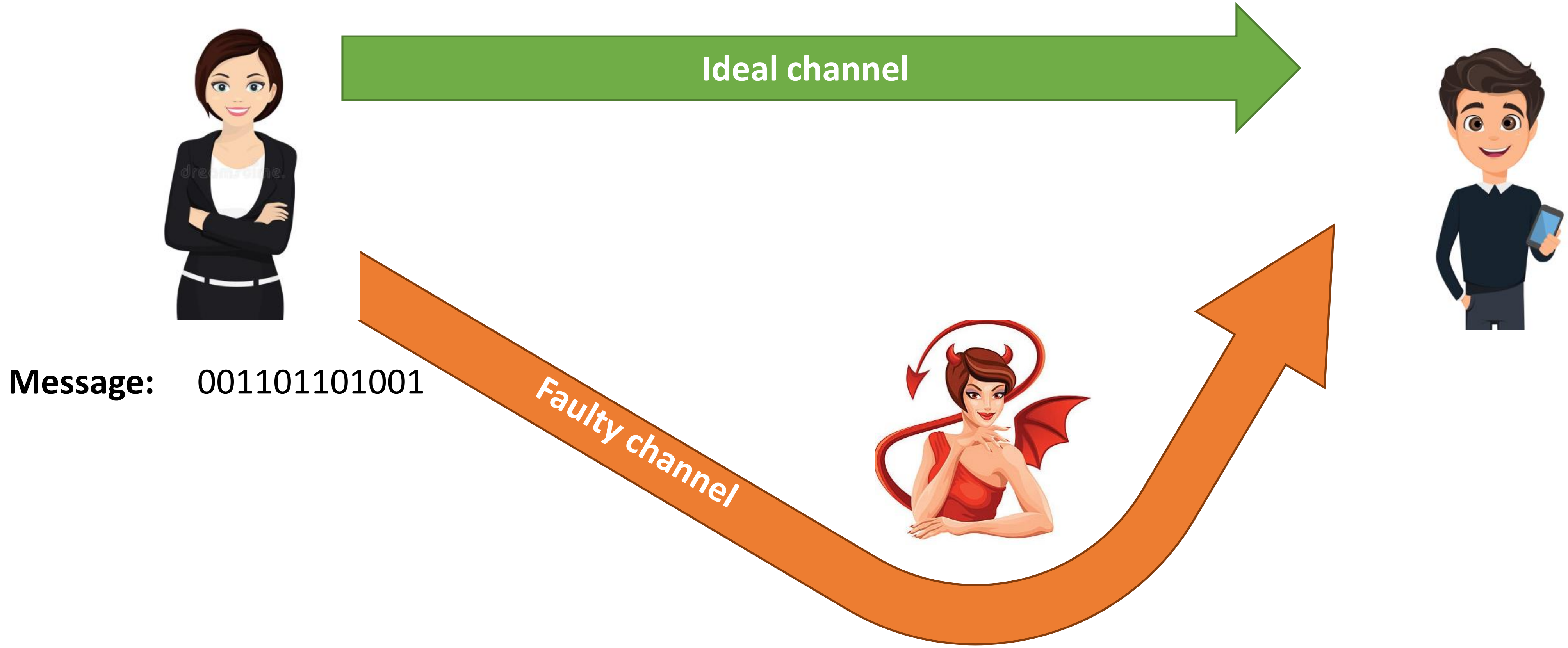
- Quantum computers/simulators
- Quantum algorithms
- Quantum machine learning

Quantum sensing:

- Quantum probes
- Quantum enhanced sensitivity

Application 1: quantum communication

Communication



Secure communication (random shared key)

Message: 0 0 1 1 0 1
Random key: 1 0 1 0 0 1 $\oplus$

Message to send: 1 0 0 1 0 0

This is random too

Faulty channel

Random key: 1 0 1 0 0 1

Message to send: 1 0 0 1 0 0 $\oplus$

Message: 0 0 1 1 0 1

Secure communication needs a random shared key

Classically there is no way to provide absolute security for key distribution

Quantum key distribution

σ_x: $\{0{:}\,|+\rangle,\qquad 1{:}\,|-\rangle\}$

σ_z: $\{0{:}\,|0\rangle,\qquad 1{:}\,|1\rangle\}$

Alice basis	Message	Bob basis	Basis matching
X	0	Z	X
X	0	Z	X
Z	1	X	X
X	1	X	√
Z	0	X	X
Z	1	Z	√
X	0	X	√

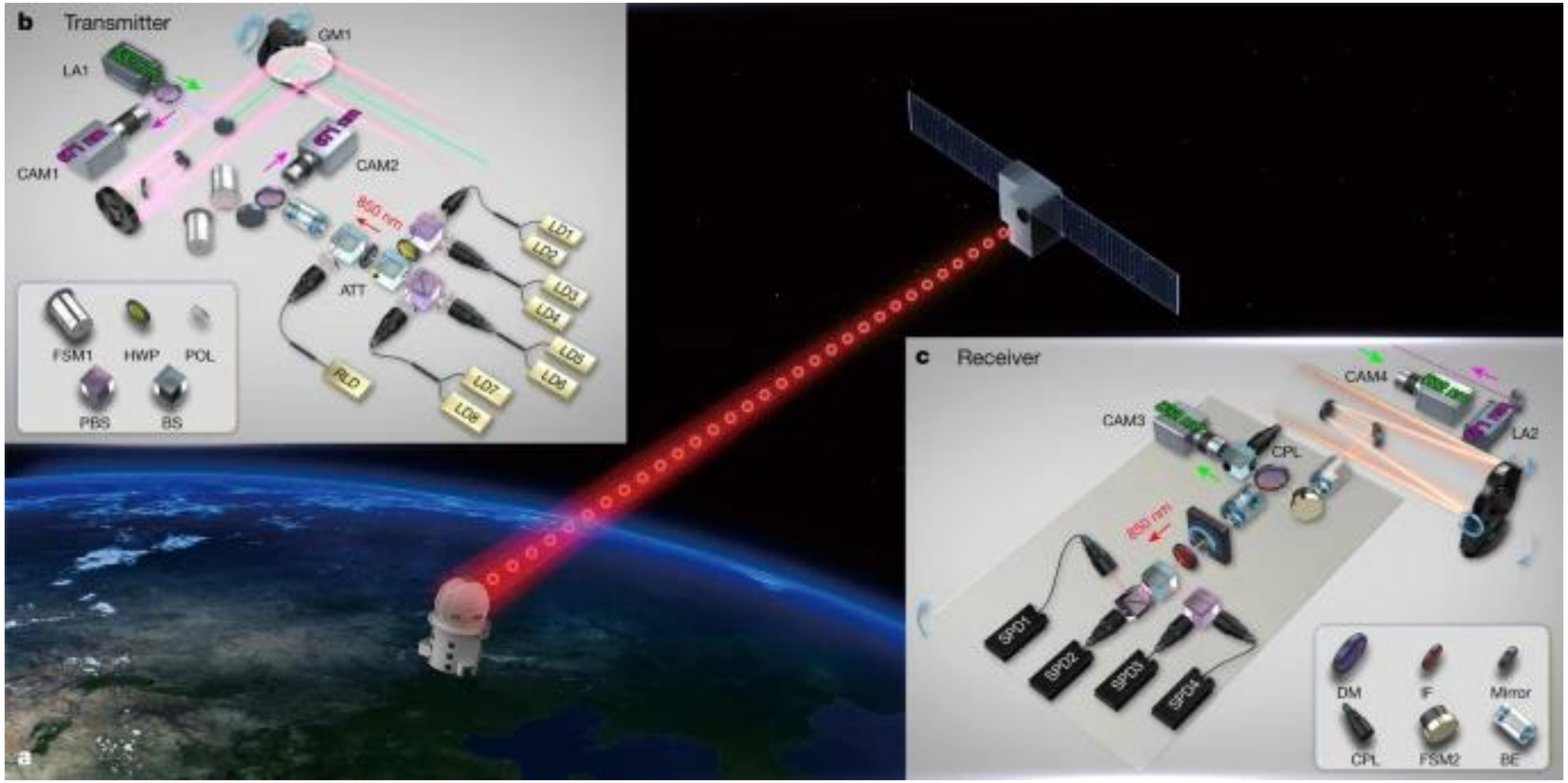
Alice sends $|-\rangle$ and Bob measures it right

Alice sends $|1\rangle$ and Bob measures it right

Alice sends $|+\rangle$ and Bob measures it right

Measures in either σ_x or σ_z

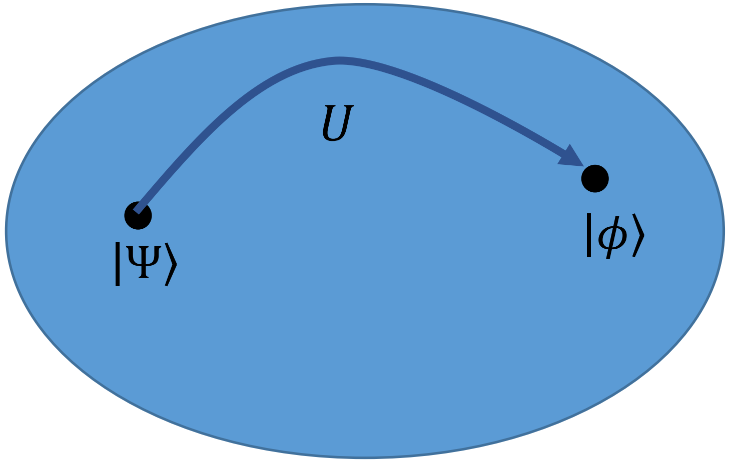
Quantum key distribution in Space



S.-K. Liao, et. al., Nature 549, 43 (2017)

Application 2: quantum computation/simulation

Quantum computer



What is a quantum computer:

- Programmable machine
- Implements any unitary operator
- Can convert any quantum state into another

Every unitary operator can be decomposed into:

- Arbitrary single qubit rotations (SU(2) rotation)

  $$U(\alpha,\beta,\gamma) = e^{-i(\alpha\sigma_x+\beta\sigma_y+\gamma\sigma_z)}$$

  $$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \qquad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- One two-qubit entangling gate

  $$U_{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad U_{CNOT}: \begin{cases} |00\rangle \to |00\rangle \\ |01\rangle \to |01\rangle \\ |10\rangle \to |11\rangle \\ |11\rangle \to |10\rangle \end{cases}$$

NISQ era

Quantum
the open journal for quantum science

HOME PUBLICATION

Quantum Computing in the NISQ era and beyond

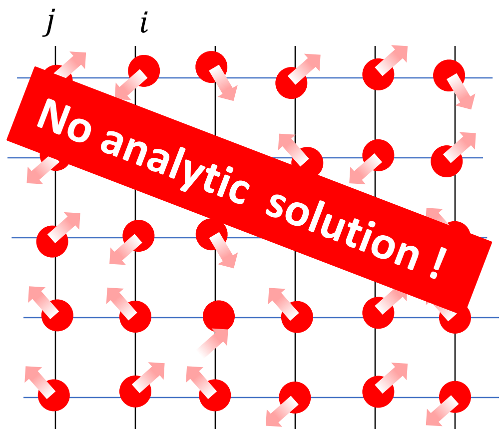
John Preskill

Institute for Quantum Information and Matter and Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena CA 91125, USA

Noisy Intermediate Scale Quantum (NISQ) devices are available.

- Noisy $\sim$ Limited coherence time ($\sim 300 - 400$ CNOT gates)
- No error correction
- Intermediate Scale (devices with $\sim 200 - 300$ qubits are available)
- Limited qubit connectivity

Quantum simulation

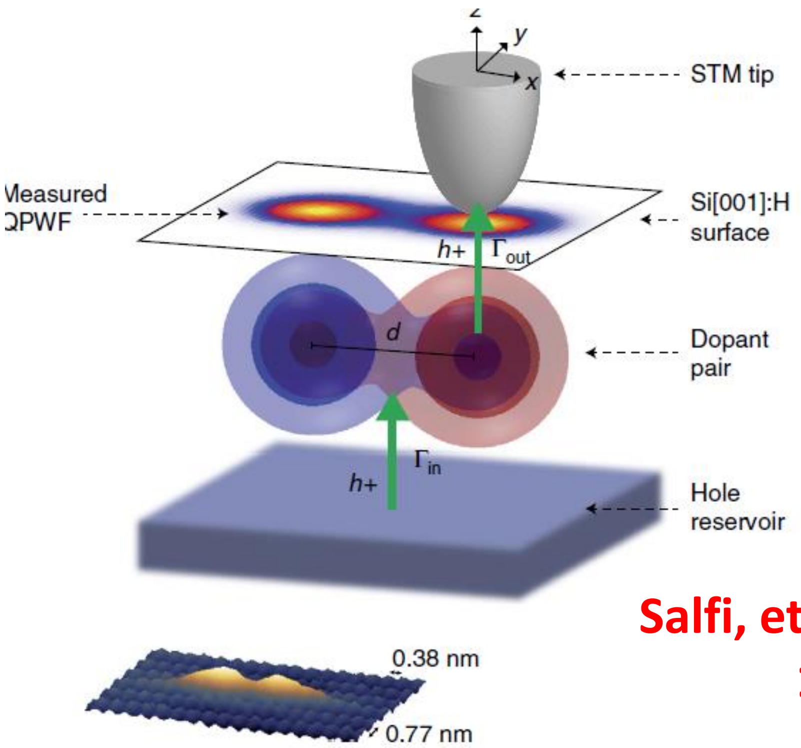


Hubbard model (High-T superconductors, …)

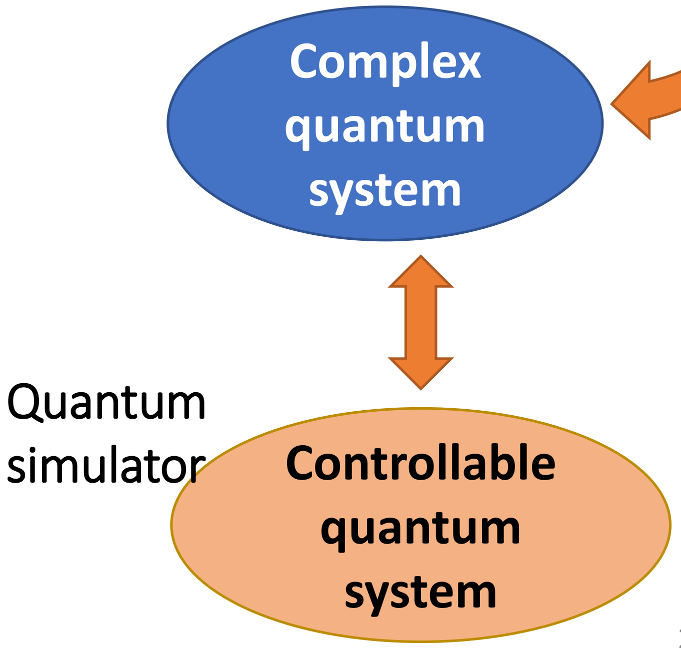
$$H = -J \sum_{<i,j>,\sigma} \hat{c}^{\dagger}{}_{i,\sigma}\, \hat{c}_{j,\sigma} + U \sum_{i} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

Classical computers cannot handle exponential problems

2^300 > Number of protons in the universe!!

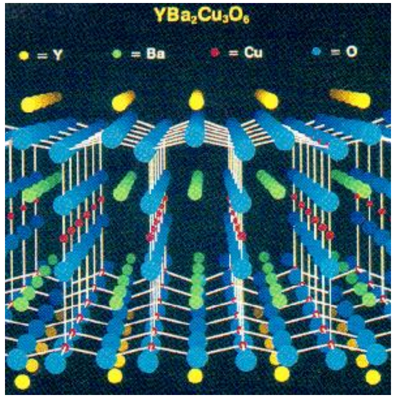


Salfi, et al, Nat. Commun. 7 11342 (2016)



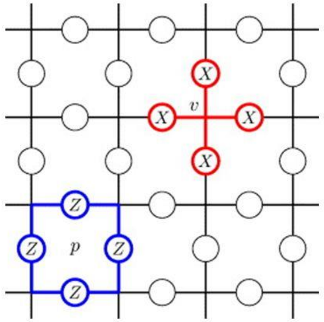
Why quantum simulators?

- Classical computers are not capable of solving quantum problems due to the exponential growth of the Hilbert space ($\sim 2^N$)



- Better controllability (e.g. high Tc superconductivity)

- Some theoretical models do not exist in nature (e.g. Kitaev toric model)



2D toric code (Kitaev quant-ph/9707021)

Qubits on links
X-stabilizers on vertices
Z-stabilizers on plaquettes

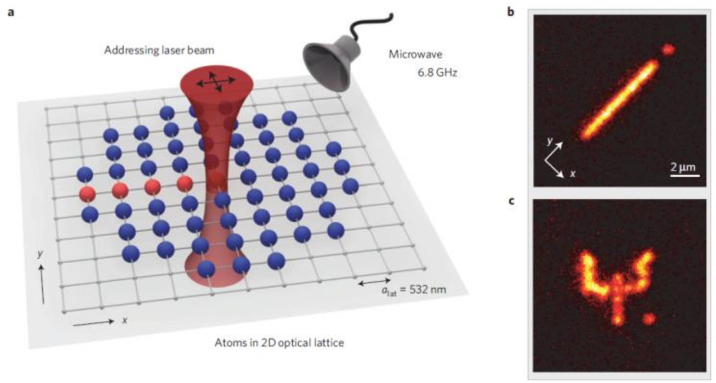
$$H = -\sum_v A_v - \sum_p B_p$$

No phase transitions
No self-correction (Alicki et al. 0810.4584)

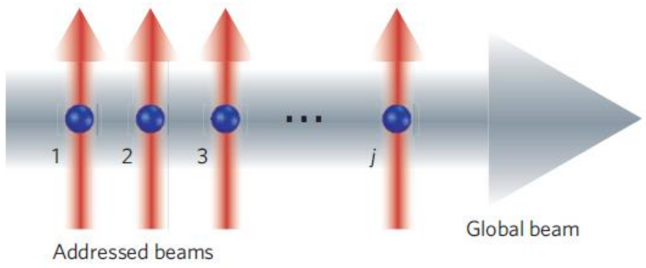
- Classical big-data problems might be solved faster on quantum computers/simulators

Quantum simulators

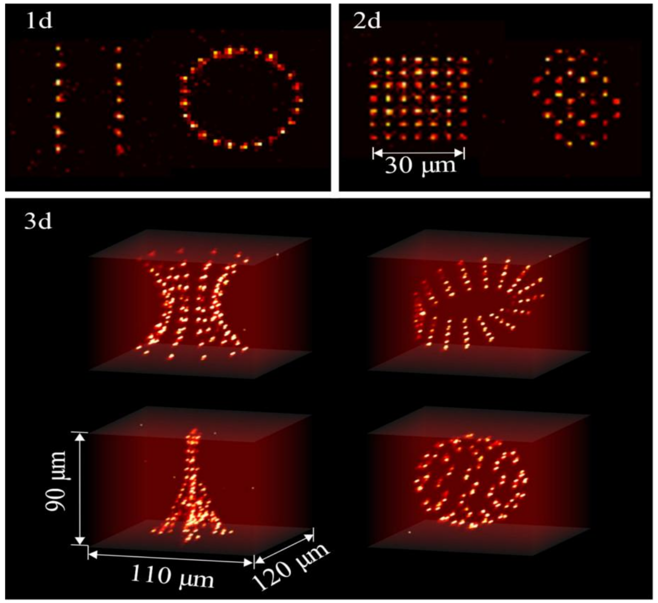
Cold atoms in optical lattices



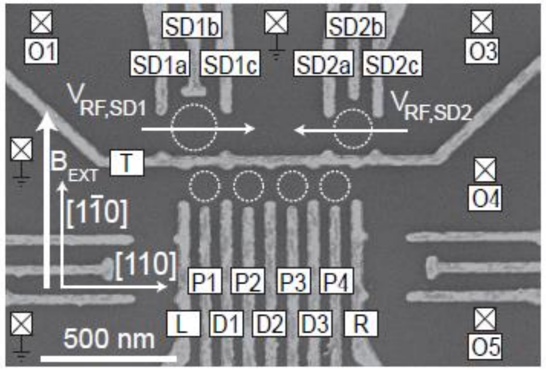
Ion traps



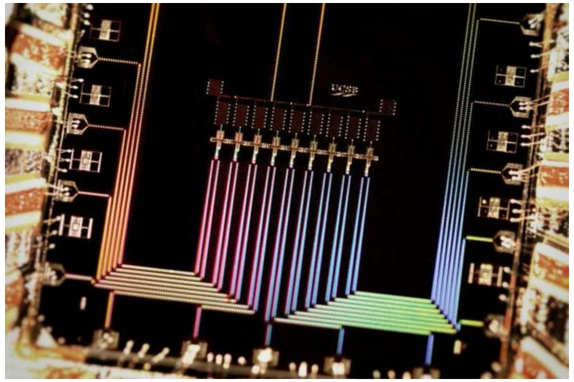
Rydberg atoms



Quantum dot arrays



Superconducting quantum simulators



NISQ era

Noisy Intermediate Scale Quantum (NISQ) devices are available.

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- Intermediate Scale (devices with $\sim 200 - 300$ qubits are available)
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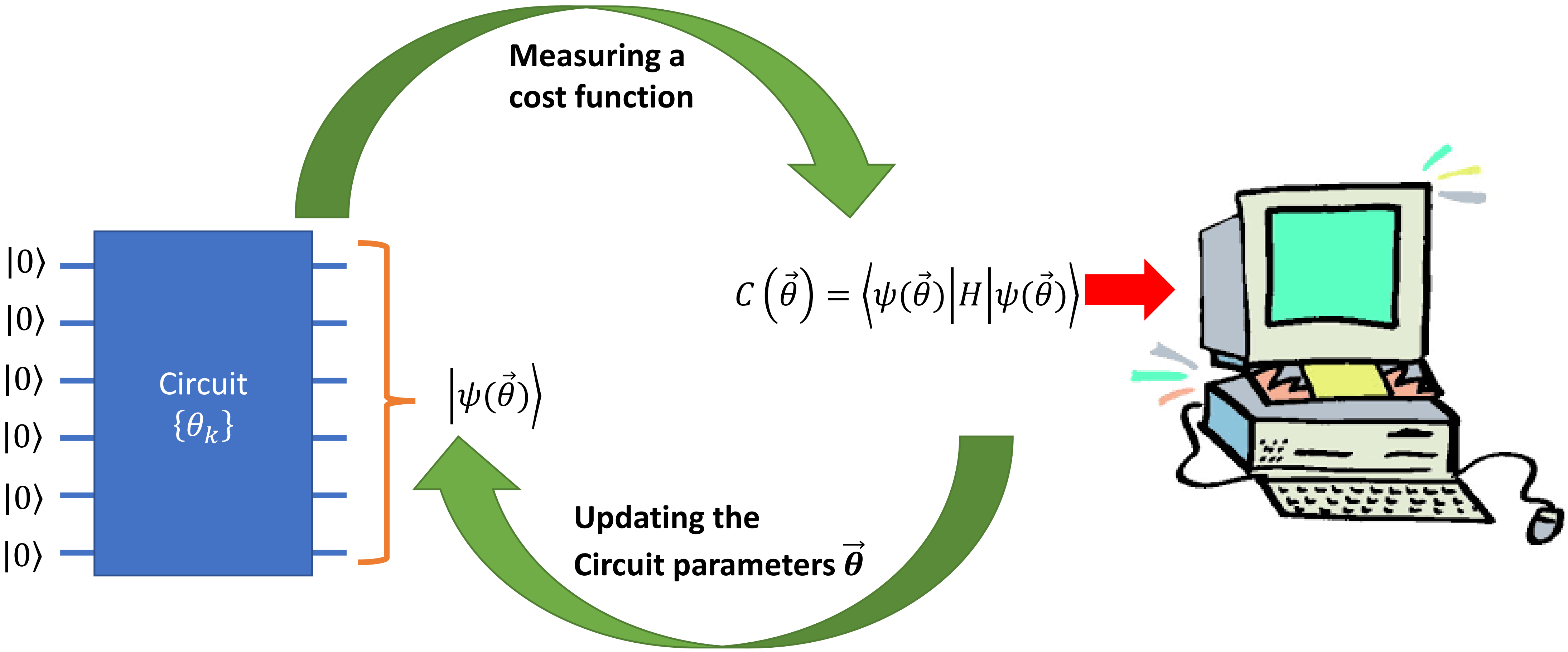
Variational Quantum Algorithms



- Complexity is divided between a quantum simulator (i.e. a shallow circuit) and a classical optimizer
- Only problems which can be written variationally can be solved
- By choosing the observable to be the Hamiltonian then the final output becomes the ground state

Resources

VQE is a hybrid algorithms using both quantum circuit and classical optimizations

1. **Quantum circuit:**
   - Number of layers or number of CNOTs
2. **Classical minimization:**
   - Number of iterations in gradient decent algorithm (convergence speed)
   - The number of parameters to optimize

Classical resources: CR=Number of parameters $\times$ Iterations

Comparison between adiabatic and VQE

	ST $1st$ $order$		ST $2nd$ $order$		VQE	
	M^*_{ad}	$\#CNOT$	M^*_{ad}	$\#CNOT$	M^*_{VQE}	$\#CNOT$
N=4	15	135	3	45	2	18
N=8	113	2373	18	594	3	63
N=10	247	6669	32	1344	3	81
N=16	770	34650	79	5451	5	225
N=20	1330	75810	132	11484	6	342

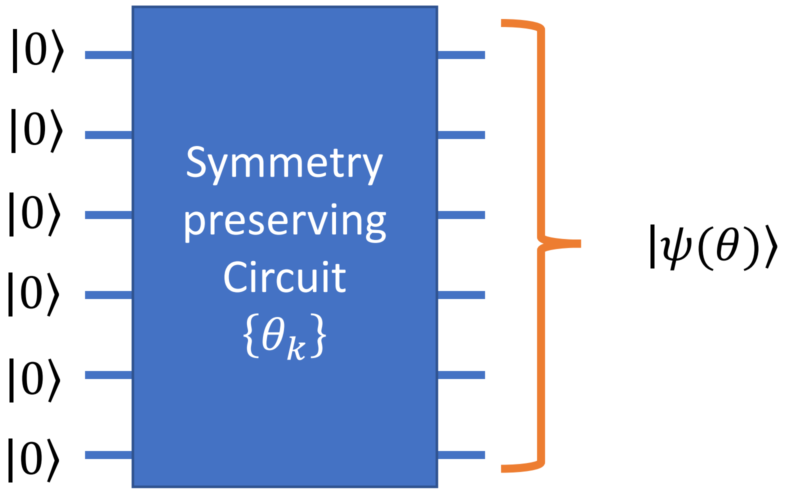
VQE can be implemented on a shallow circuit

Implementing symmetries in VQE

$[H, S] = 0$ $\Rightarrow$ H and S have common eigenvectors

1- Penalizing the cost function: $C(\theta) = \langle H \rangle + (\langle S \rangle - S_{tar})^2$

2- Implementing the symmetries in the quantum circuit so that $|\psi(\theta)\rangle$ naturally conserves the symmetry

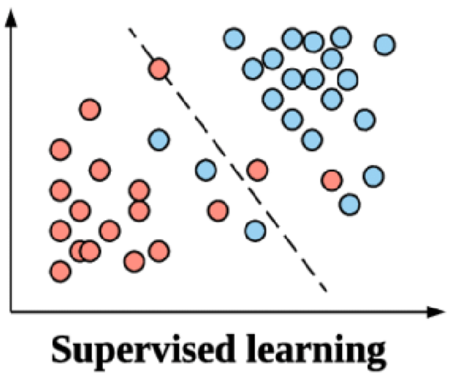


The circuit output naturally conserves the symmetry: $\langle \psi(\theta)|S|\psi(\theta)\rangle = S_{tar}$

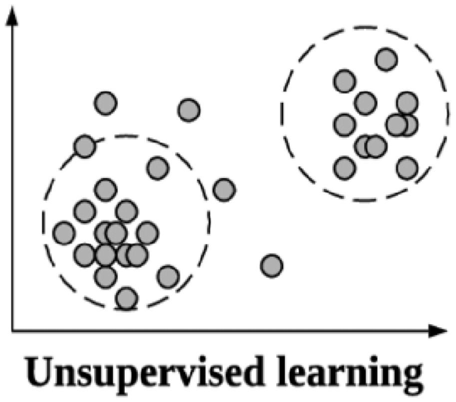
Implementation of symmetries in the hardware is more resource (both quantum and classical) efficient

Quantum machine learning

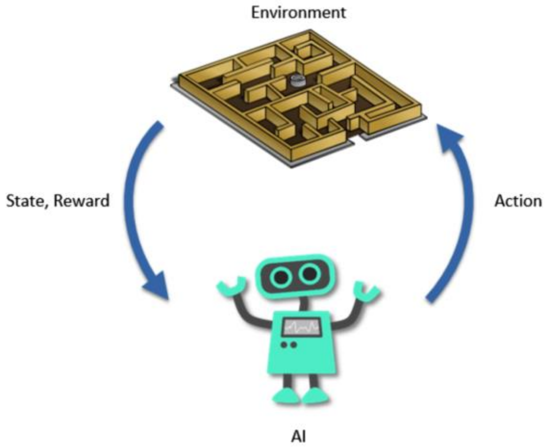
Machine learning



1- Supervised learning: To predict the label of an unknown input data, e.g. classification.



2- Unsupervised learning: To group the inputs according to their similarities, e.g. clustering.



3- Reinforcement learning: There is no data to train. The system learns by rewarding (or punishing) the desired (undesired) outputs.

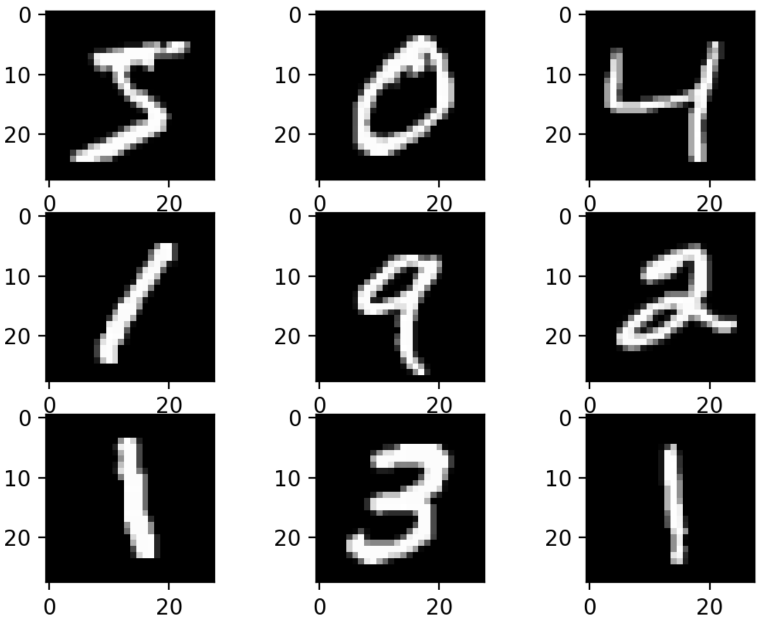
Classification Problems

Thanks to the Internet, we have loads of labeled data

Classification is one of the main machine learning algorithms

Cats vs Dogs (2 classes: binary)

Handwriting digits (10 classes)



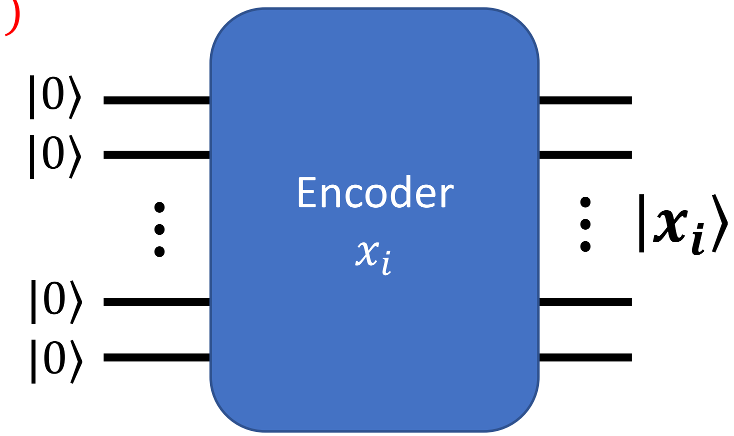
Quantum classifiers

- Can quantum computers solve classification problems?
- If so, can they outperform classical classifiers?
- Datasets can be either classical (handwriting) or quantum (ground state of a Hamiltonian).
- For quantum datasets it is likely that quantum computers are useful.
- For classical datasets it is still an open problem whether quantum computers can provide any advantage.

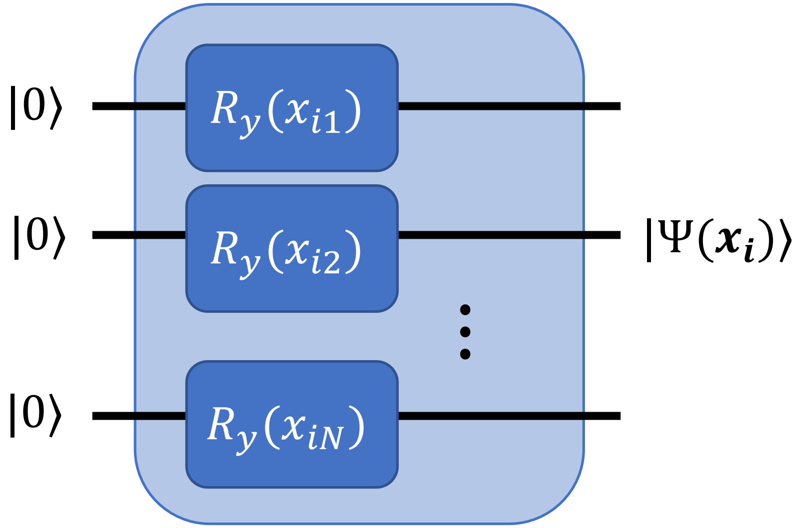
Encoding classical datasets

Amplitude encoding: It provides an exponential advantage. Number of qubit=$\log(N)$

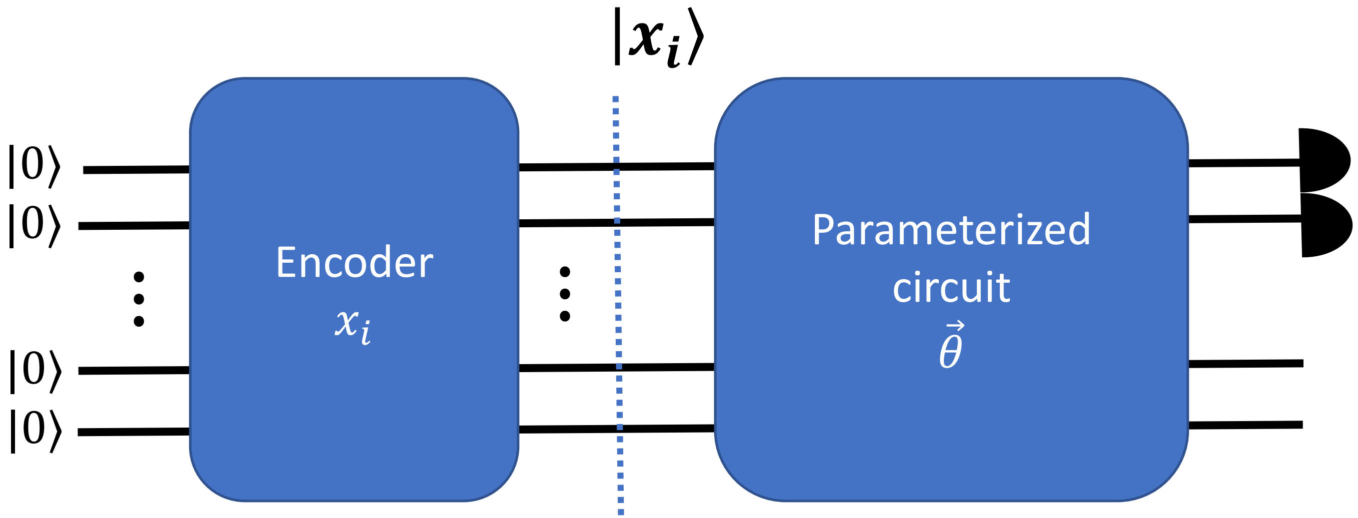
Input data: $\boldsymbol{x_i} = (x_{i1}, x_{i1}, \ldots, x_{iN})$ $\Rightarrow$ $$\boldsymbol{x}_i \rightarrow |\boldsymbol{x}_i\rangle = \frac{1}{||\boldsymbol{x}_i||} \sum_{j=1}^{N_\mathrm{f}} x_{ij} |j\rangle$$



Rotation encoding: It is easy for experiments but without advantage in scaling. Number of qubit=N



Variational classifiers

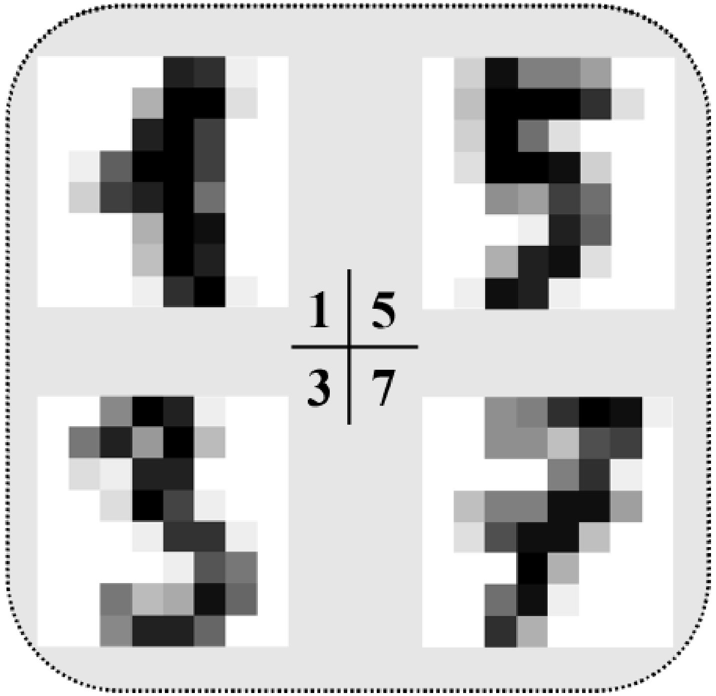


Measurement outcome	Class	Probability
00	0	P_{00}
01	1	P_{01}
10	2	P_{10}
11	3	P_{11}

- The largest probability determines the class
- For probability vector $P_i = (P_{i1}, P_{i2}, \ldots)$ The loss function can be defined as: $Y_i = (0,0,1,0)$

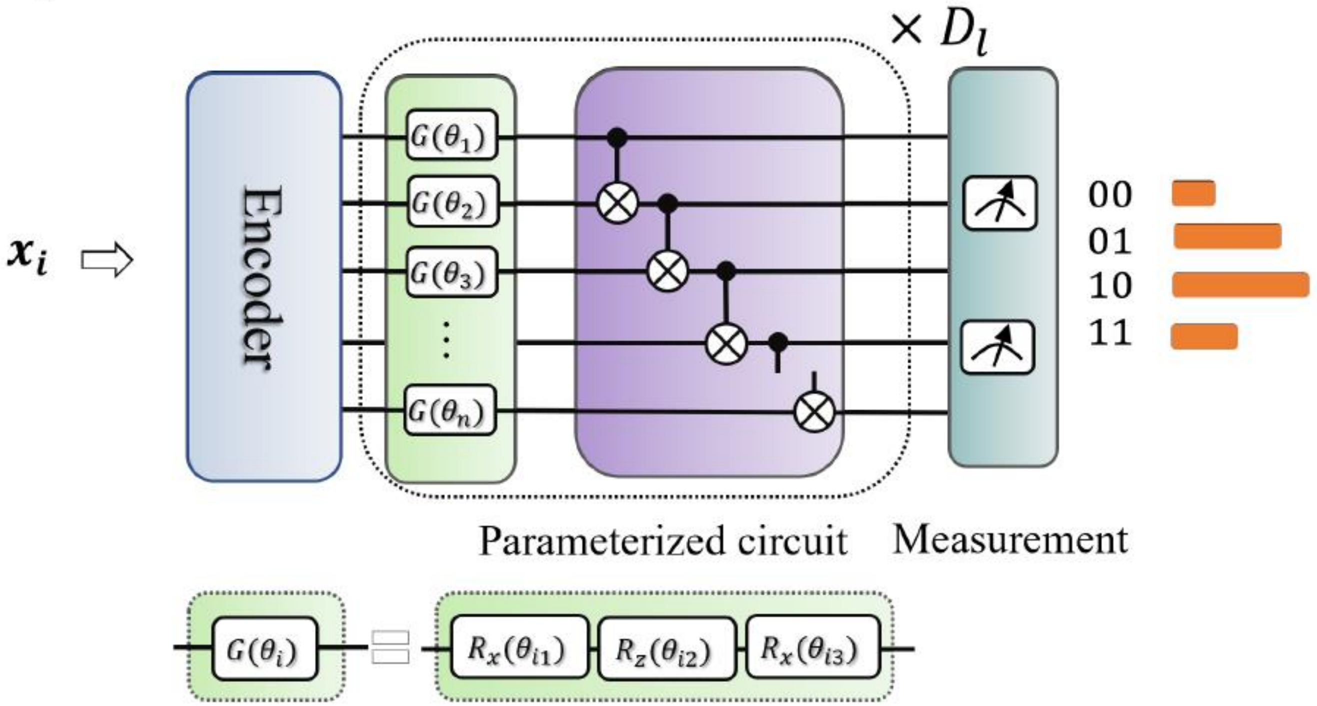
$$\mathcal{L}\left(\vec{\theta}\right) = -\sum_i Y_i^T \log(P_i)$$

MNIST dataset (odd digits)



- Each image is a 8×8 pixels (64 features)
- Four different classes (odd numbers)

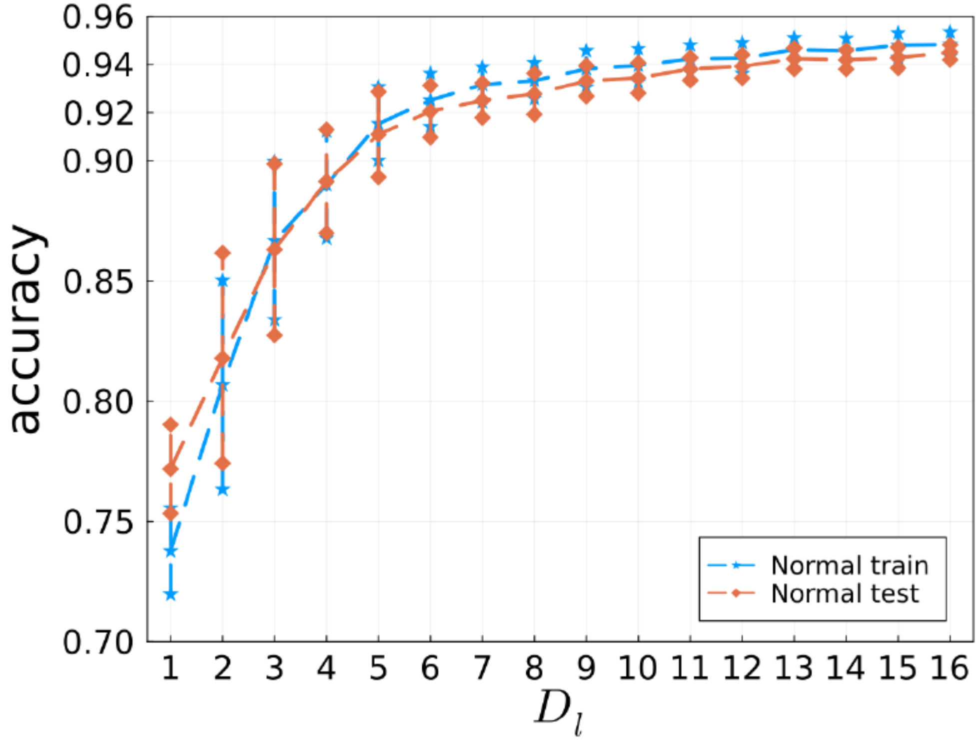
Quantum circuit



We use amplitude encoding but the protocol works for rotation encoding too

Error rate: $e = \frac{1}{M_s} \sum_{i=1}^{M_s} \mathbb{I}(f(\boldsymbol{x}_i) \neq y_i)$ Where $\mathbb{I}(\text{True}) = 1$ and $\mathbb{I}(\text{False}) = 0$

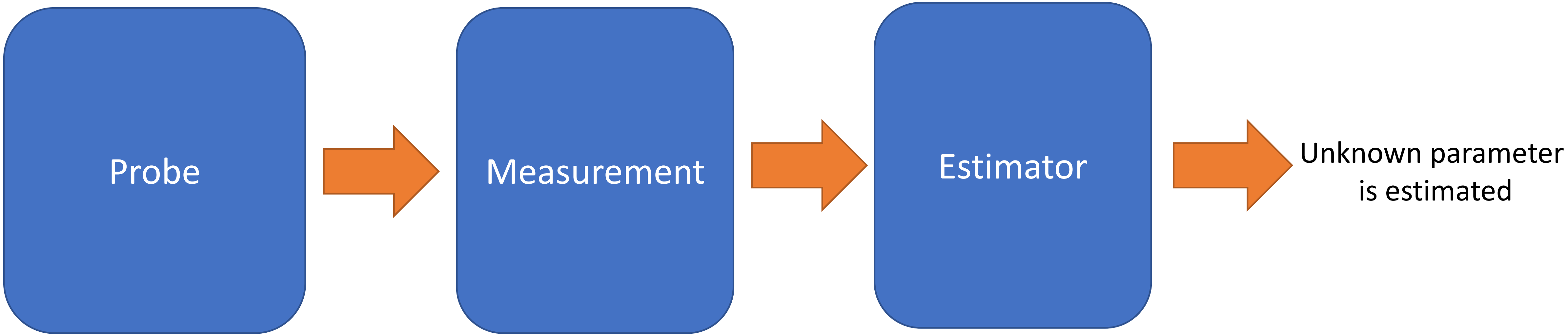
The effect of layers



- In a noise-free quantum computer the accuracy increases by increasing the layers
- Training and test errors remain close to each other showing the absence of over-fitting

Application 3: quantum sensing

Sensing procedure



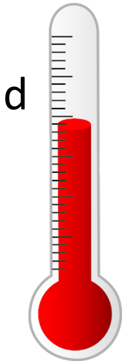
The probe, measurement basis and estimators should be optimal

Classical Fisher information

One can measure a random variable X for measuring an unknown parameter θ

X can be the height of the thermometer's liquid

θ can be temperature



With the probability of $P_x(\theta)$ the height is measured to be x

Cramer-Rao inequality: $$\delta\theta^2 = \langle(\theta_{est} - \theta_{real})^2\rangle \geq \frac{1}{MF(\theta)}$$

M is the number of repetition

Fisher information: $$F(\theta) = \sum_x P_x(\theta)\left(\frac{d\log(P_x(\theta))}{d\theta}\right)^2 = \sum_x \frac{1}{P_x(\theta)}\left(\frac{d\,P_x(\theta)}{d\theta}\right)^2$$

Larger Fisher information implies better sensitivity

Quantum Fisher information

$\rho(\theta) \Rightarrow$ Estimate θ

1. One can choose both measurement basis and the estimator

2. For any choice of measurement we have projectors: $\{M_k\}$

3. The probability of each outcome is: $p_k(\theta) = tr[\rho(\theta)M_k]$

4. Then quantum Fisher information: $F_q(\theta) = Max_{\{Mk\}}[F_c(\theta)]$ $\Rightarrow$ $F_q \geq F_c$

For a classical probe of size N, at best one can achieve: $F_C = N$

Quantum Fisher Information

$$\rho(\theta) = \sum_k q_k(\theta)|\psi_k(\theta)\rangle\langle\psi_k(\theta)| \xrightarrow{\text{SLD}} \frac{\partial\rho(\theta)}{\partial\theta} = \frac{L(\theta)\rho(\theta) + \rho(\theta)L(\theta)}{2}$$

$$\Rightarrow \quad F_q(\theta) = Tr\left[\rho(\theta)L^2(\theta)\right]$$

For pure states: $\rho(\theta) = |\Psi(\theta)\rangle\langle\Psi(\theta)|$ $\Rightarrow$ $F_q(\theta) = 4\left[\langle\partial_\theta\Psi(\theta)|\partial_\theta\Psi(\theta)\rangle - \left|\langle\Psi(\theta)|\partial_\theta\Psi(\theta)\rangle\right|^2\right]$

- Optimal measurement basis will be the eigenstates of $L(\theta)$
- The optimal basis might depend on the unknown parameter θ
- Super linear scaling (i.e. quantum enhanced sensitivity) might be possible

Quantum sensing

$$H = B\sum_k \sigma_k^z \qquad |\Psi_0\rangle = \frac{|0,0,\dots,0\rangle + |1,1,\dots,1\rangle}{\sqrt{2}} \Rightarrow |\Psi_t\rangle = e^{-iHt}|\Psi_0\rangle = \frac{|0,0,\dots,0\rangle + e^{-iNBt}|1,1,\dots,1\rangle}{\sqrt{2}}$$

Proper measurement

$$\begin{cases} P_{x+} = cos^2(NBt) \\ P_{x-} = sin^2(NBt) \end{cases}$$

Fisher information: $F(B) = 4N^2t^2$

Precision

$$\delta B^2 \geq \frac{1}{4N^2t^2}$$

Heisenberg limit

V. Giovannetti, S. Lloyd, and L. Maccone, Science 306, 1330 (2004).
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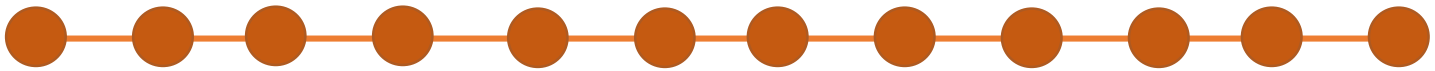
Criticality enhanced sensing



$H = \theta H_1 + H_2$ $\Rightarrow$ $|GS(\theta)\rangle$ $\Rightarrow$ Can one estimate θ directly from the ground state?

- Near criticality ($\theta \approx \theta_c$): $F_q(\theta) \sim N^{2/d\upsilon}$
- Far from criticality: $F_q(\theta) \sim N|\theta - \theta_c|^{d\upsilon - 2}$

For Ising in transverse field ($d = 1, \upsilon = 1$) around criticality: $F_q(\theta) \sim N^2$ Heisenberg limit

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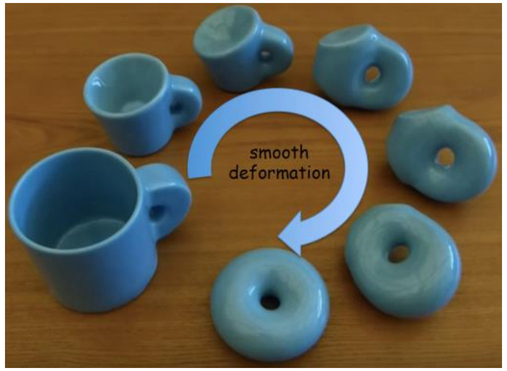
Quantum enhanced sensitivity

What is responsible for quantum enhanced sensitivity?

- ❑ Scale invariance
- ❑ Symmetry-breaking
- ❑ Long-range entanglement/correlations
- ❑ Gap closing

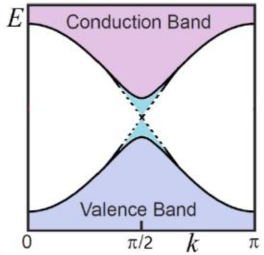
Symmetry protected topological Systems

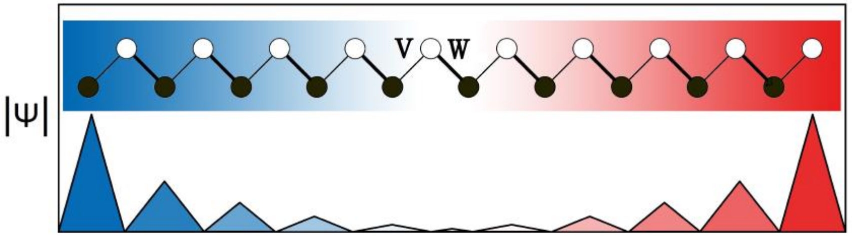


Topological properties don't change under continuous deformations.

Phase transitions in such systems are not captured by Landau's theory:

- They are captured by of a global order parameter
- There is no symmetry breaking
- There is no long-range entanglement
- The gap closing still remains valid
- Zero energy edge states emerge in the system





Around the critical point the edge states show Heisenberg precision

This shows that gap closing is the key feature for quantum enhanced sensitivity

Summary

Quantum technologies may change our lives in a fundamental way in coming decades

Quantum features (such superposition and measurement) can be exploited for surpassing the performance of classical devices.

The most important aspects of quantum technologies are:

- Quantum communications
- Quantum simulation
- Quantum sensing