# Quantum

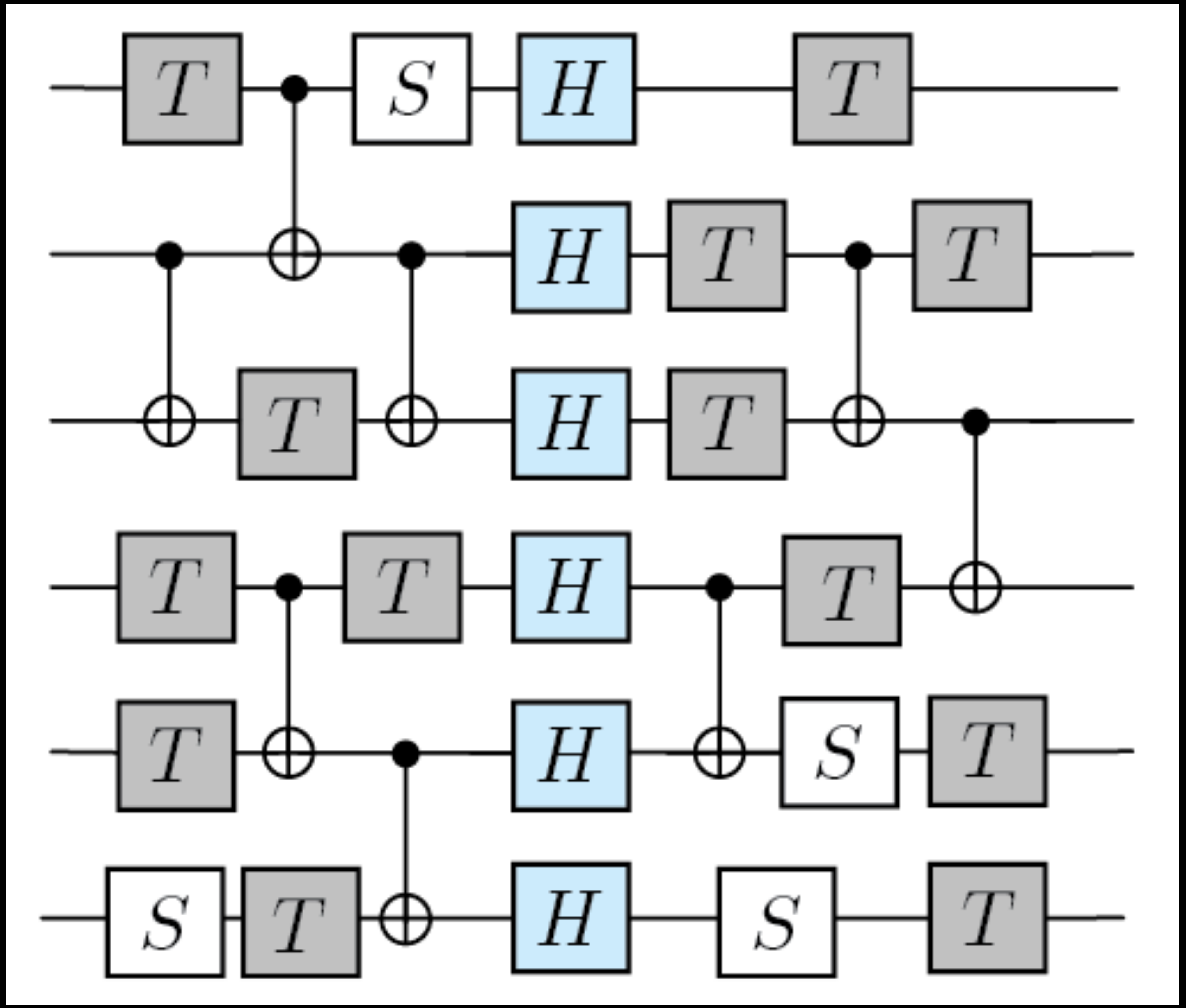
## Machine Learning

# معرفی یادگیری ماشین کوانتومی

فرهاد فضیله

۱۴۰۲/۶/۱

# فهرست

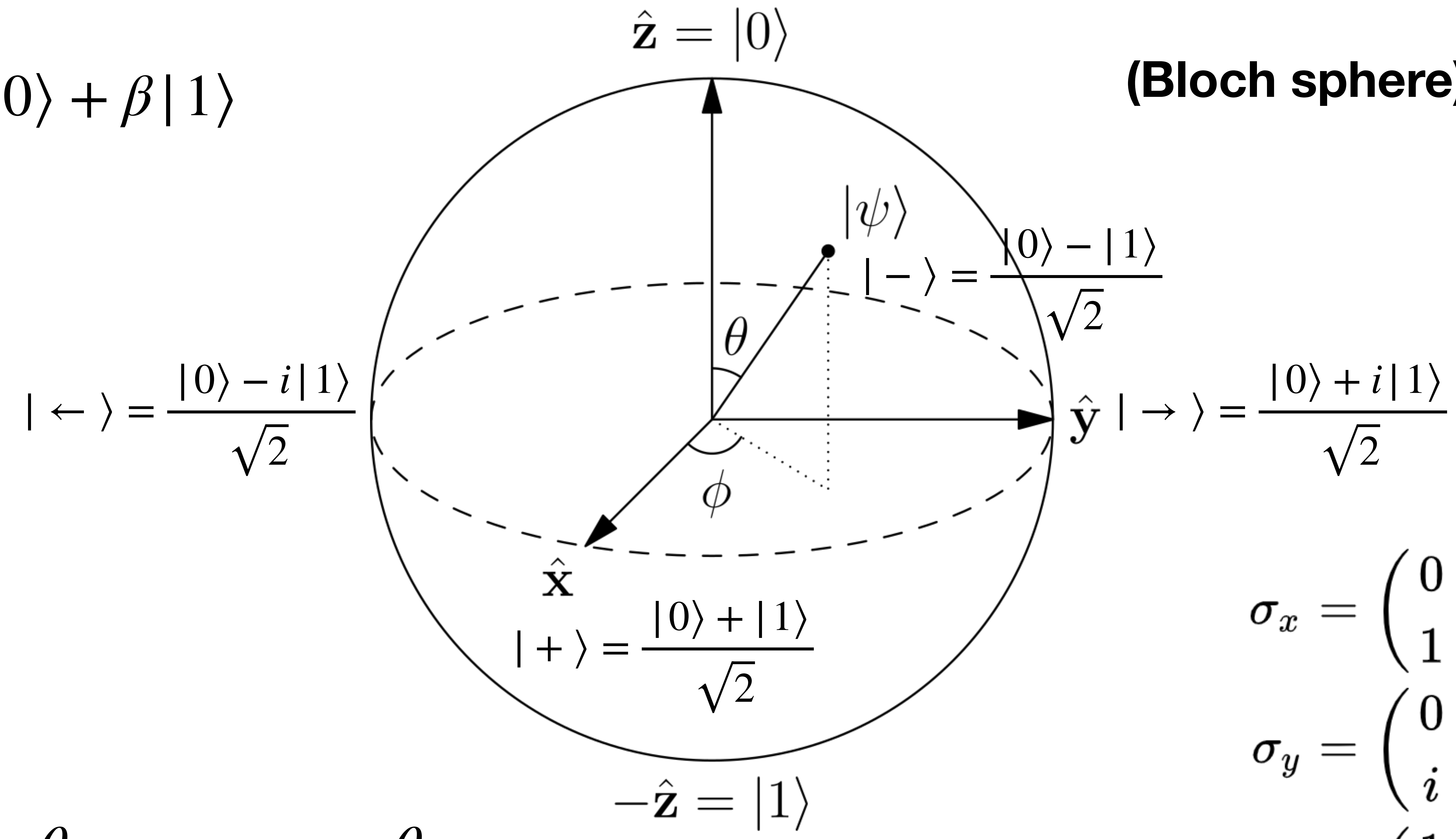



- معرفی کامپیوترهای کوانتومی
- معرفی IBM Quantum Composer
- خلاصه‌ای از یادگیری ماشین کلاسیک
- یادگیری ماشین کوانتومی

# مروری بر مبانی نظری دروازه‌های کوانتومی

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

کره بلاخ (Bloch sphere)




$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = X$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = Y$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Z$$

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

# مروری بر مبانی نظری دروازه‌های کوانتومی

$$|0\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad |1\rangle := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle := \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}; \quad |\alpha|^2 + |\beta|^2 = 1.$$

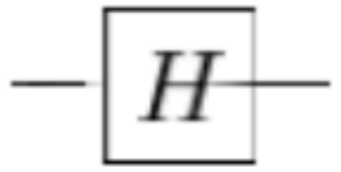
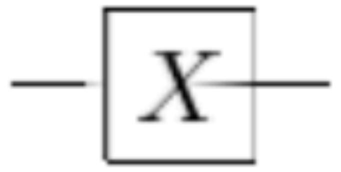
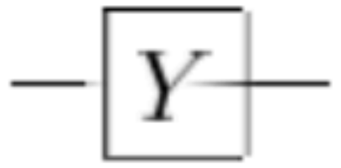
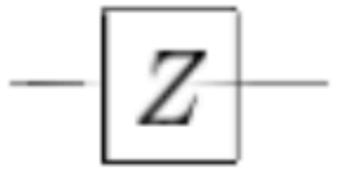
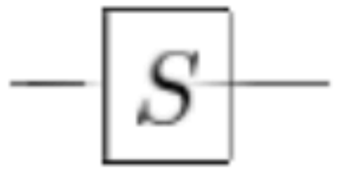
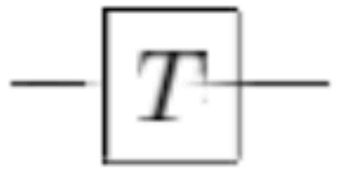
$$|00\rangle := \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad |01\rangle := \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \quad |10\rangle := \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \quad |11\rangle := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

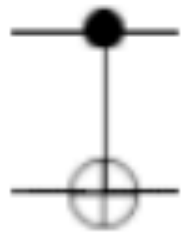
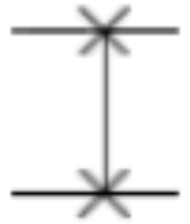
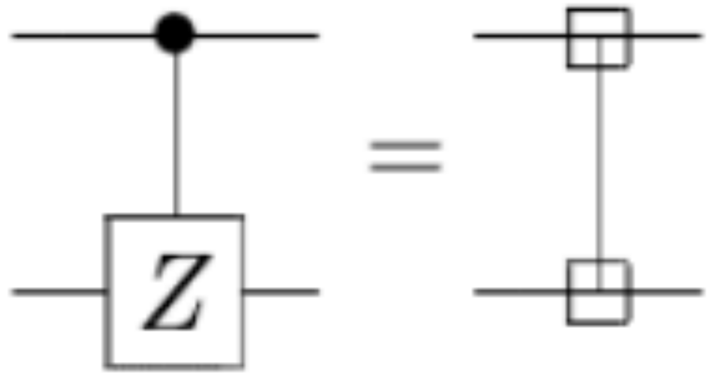
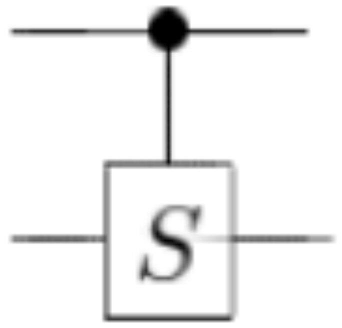
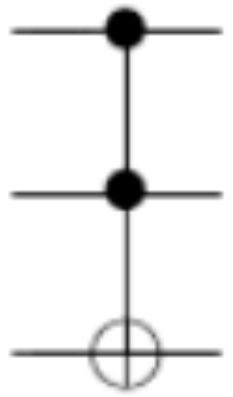
$$CNOT := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$CNOT\,\frac{|00\rangle + |10\rangle}{\sqrt{2}} = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$CNOT|00\rangle = |00\rangle$, $CNOT|01\rangle = |01\rangle$, $CNOT|10\rangle = |11\rangle$, and $CNOT|11\rangle = |10\rangle$

Quantum Gates:

| | | |
|---|---|---|
| Hadamard |   | $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ |
| Pauli-$X$ |   | $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ |
| Pauli-$Y$ |   | $\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$ |
| Pauli-$Z$ |   | $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ |
| Phase |   | $\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$ |
| $\pi/8$ |   | $\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$ |

| | | |
|---|---|---|
| controlled-NOT |  | $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$ |
| swap |  | $\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$ |
| controlled-$Z$ |   | $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}$ |
| controlled-phase |   | $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&i\end{bmatrix}$ |
| Toffoli |  | $\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}$ |

# Quantum Gate sets

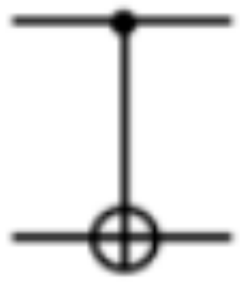
- Clifford Gate Set
  - Hadamard gate


$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

  - S gate

$$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$

  - CNOT gate



$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$$

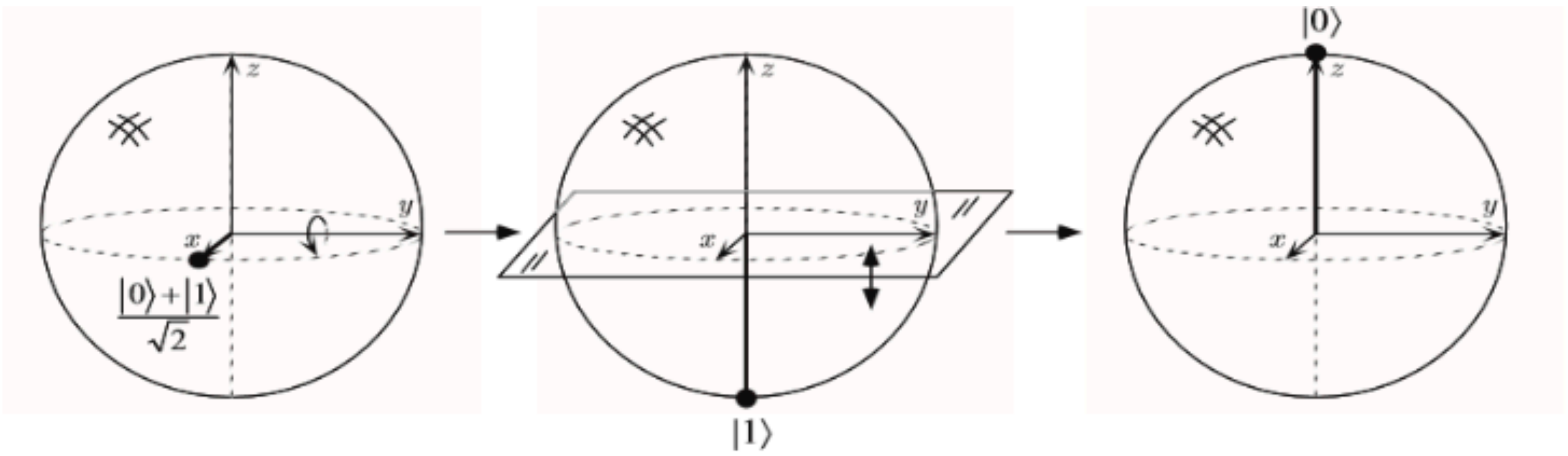
Universal Gate Set

Hadamard gate

T gate

$$\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$$

CNOT gate




# Hadamard Gate

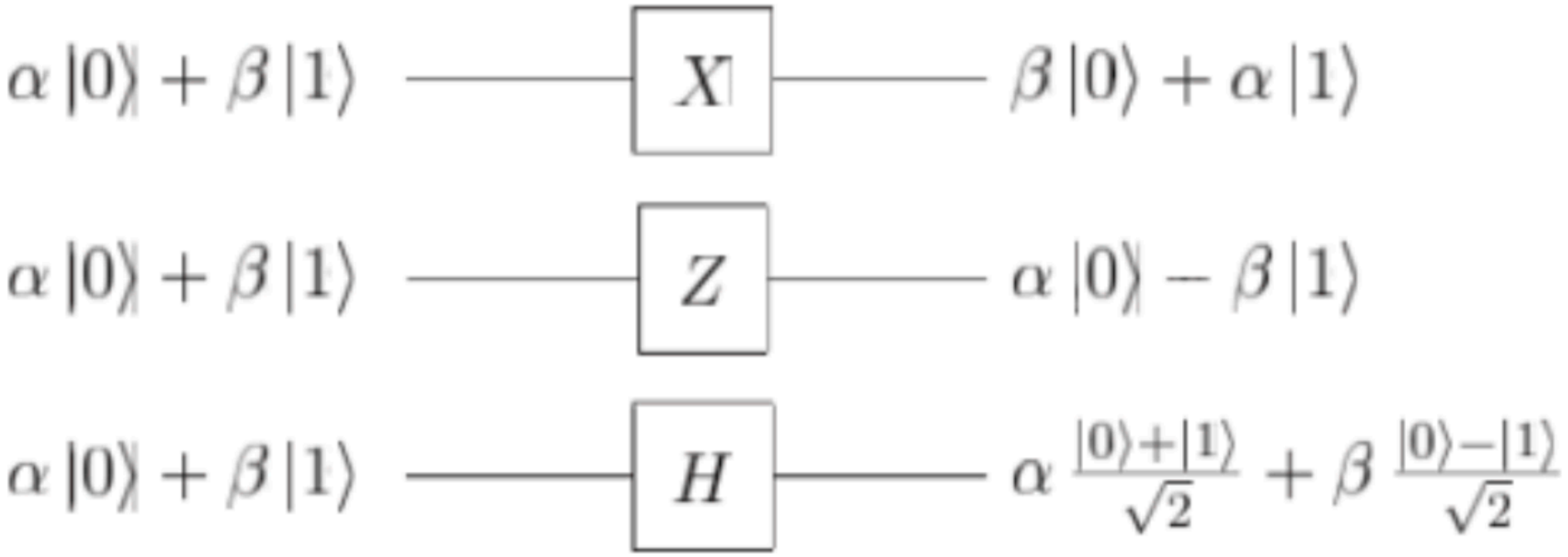
$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |+\rangle,$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = |-\rangle.$$

, $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$,

$|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$.



# مروری بر مبانی نظری دروازه‌های کوانتومی

## ماتریس چگالی (Density Matrix):

**حالت خالص (pure state):** $|\psi\rangle \qquad \rho = |\psi\rangle\langle\psi|$

**حالت مخلوط (mixed state):**

$$\left.\begin{array}{l} p_1 : |\psi_1\rangle \\ p_2 : |\psi_2\rangle \end{array}\right\} \quad \rho = p_1|\psi_1\rangle\langle\psi_1| + p_2|\psi_2\rangle\langle\psi_2|$$

$$\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i| \qquad tr(\rho) = \sum_i p_i = 1$$

$tr(\rho^2) = 1$ **pure state**

$tr(\rho^2) < 1$ **mixed state**

**Eigenvalues of ρ ?**

**pure state:** $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$$\rho = |\psi\rangle\langle\psi| = \frac{1}{2}(|0\rangle + |1\rangle)(\langle 0| + \langle 1|) = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

$$\rho^2 = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

**mixed state:**

$$\left.\begin{array}{l} p_1 = 1/2 : |0\rangle \\ p_2 = 1/2 : |1\rangle \end{array}\right\} \quad \rho = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \qquad \rho^2 = \begin{pmatrix} 1/4 & 0 \\ 0 & 1/4 \end{pmatrix}$$

# مروری بر مبانی نظری دروازه‌های کوانتومی

**ماتریس چگالی کاهش یافته (Reduced Density Matrix):**

state space is the tensor product $H_A \otimes H_B$ of Hilbert spaces

$$\rho^A = \mathrm{Tr}_B\, \rho$$

$$|\psi\rangle = \frac{1}{2}(|00\rangle + |11\rangle) \qquad \rho = \frac{1}{2}(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|)$$

$$\rho^A = Tr_B \rho = \langle 0|_B \rho |0\rangle_B + \langle 1|_B \rho |1\rangle_B = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

# قوی ترین مجموعه‌های کامپیوترهای کوانتومی جهان

1- IBM ——> QASM , Qiskit platforms

2- Google ——> Tensorflow Quantum (TFQ) , Cirq

IBM Q System One (Fraunhofer)

3- ETH Zurich ——> ProjectQ

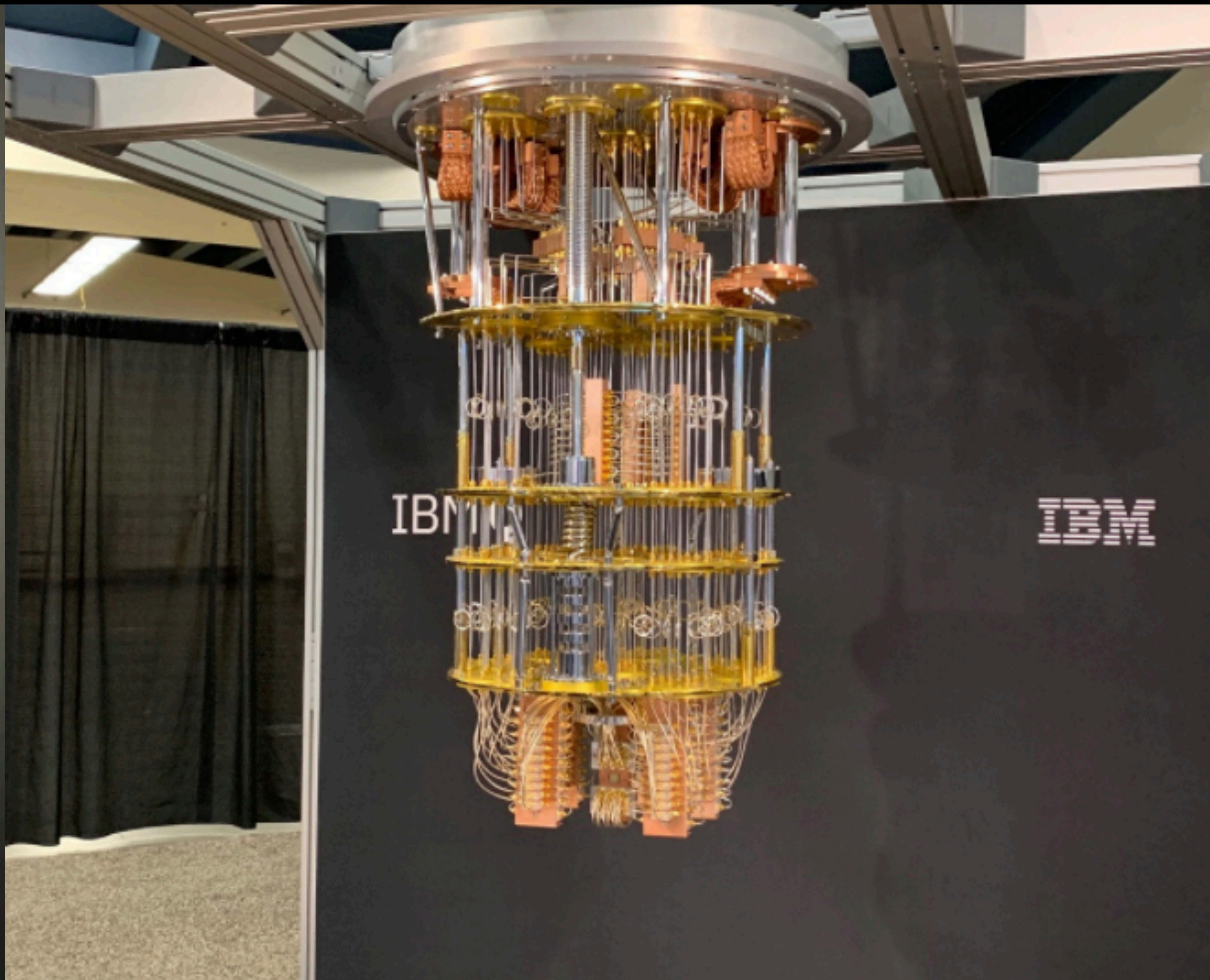
4- Perceval (France) ——> photonic quantum computer

5- Microsoft ——> Q#

6- Xanadu (Toronto) ——> Strawberry Field platform ——> Pennylane

7- NASA Google ——> D-wave Quantum Annealer

# IBM Quantum Computer

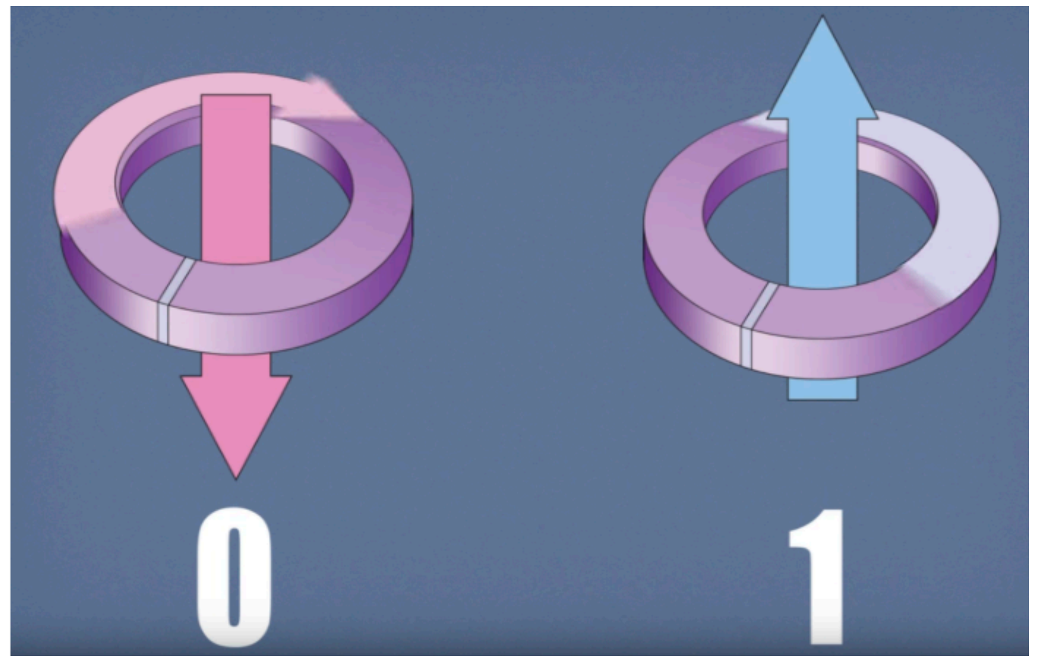


# D-Wave System

For the D-Wave system, the Hamiltonian may be represented as

$$\mathcal{H}_{ising} = \underbrace{-\frac{A(s)}{2}\left(\sum_i \hat{\sigma}_x^{(i)}\right)}_{\text{Initial Hamiltonian}} + \underbrace{\frac{B(s)}{2}\left(\sum_i h_i \hat{\sigma}_z^{(i)} + \sum_{i>j} J_{i,j}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)}\right)}_{\text{Final Hamiltonian}}$$

where $\hat{\sigma}_{x,z}^{(i)}$ are Pauli matrices operating on a qubit $q_i$, and $h_i$ and $J_{i,j}$ are the qubit biases and coupling strengths.[1]



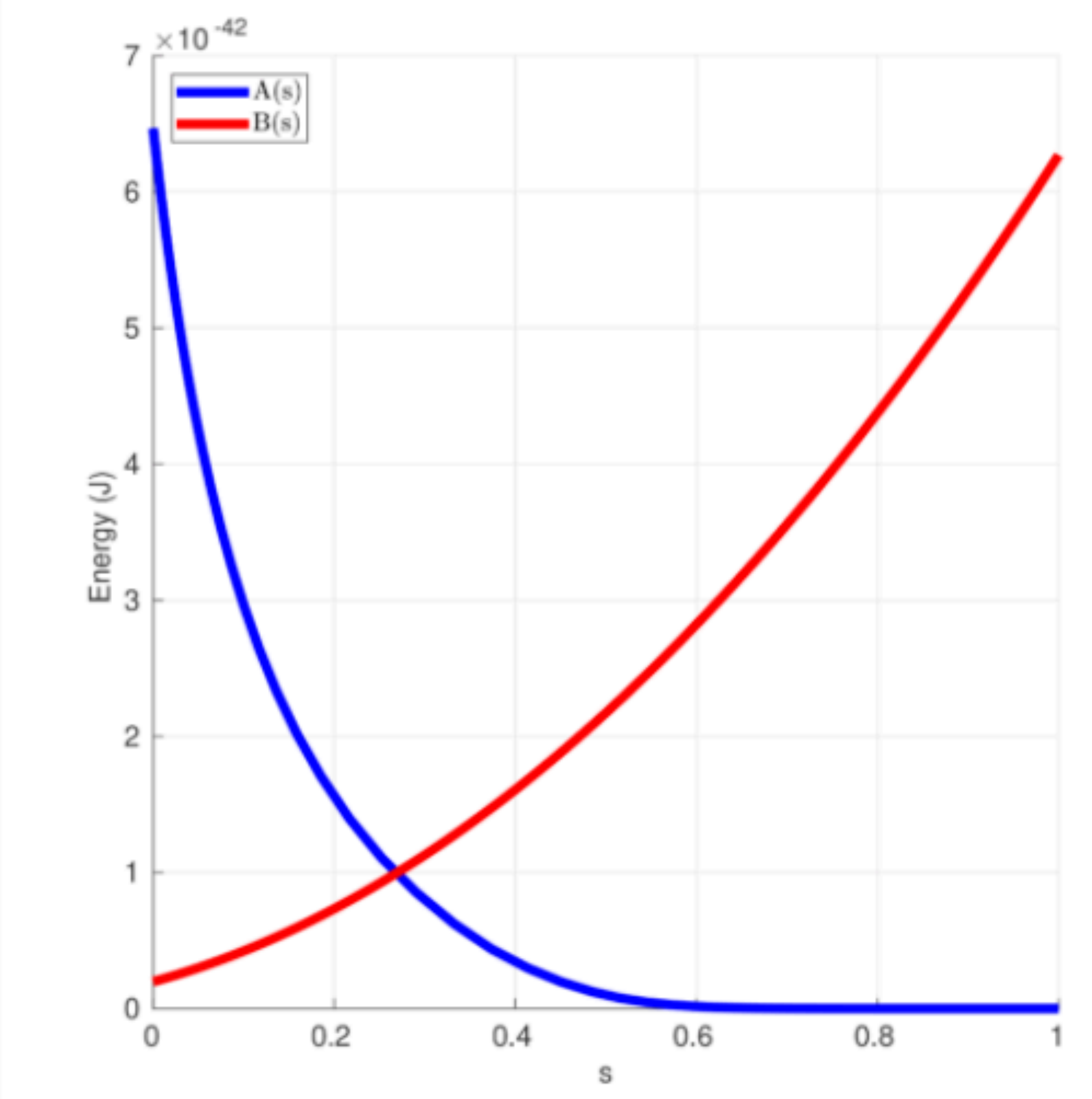
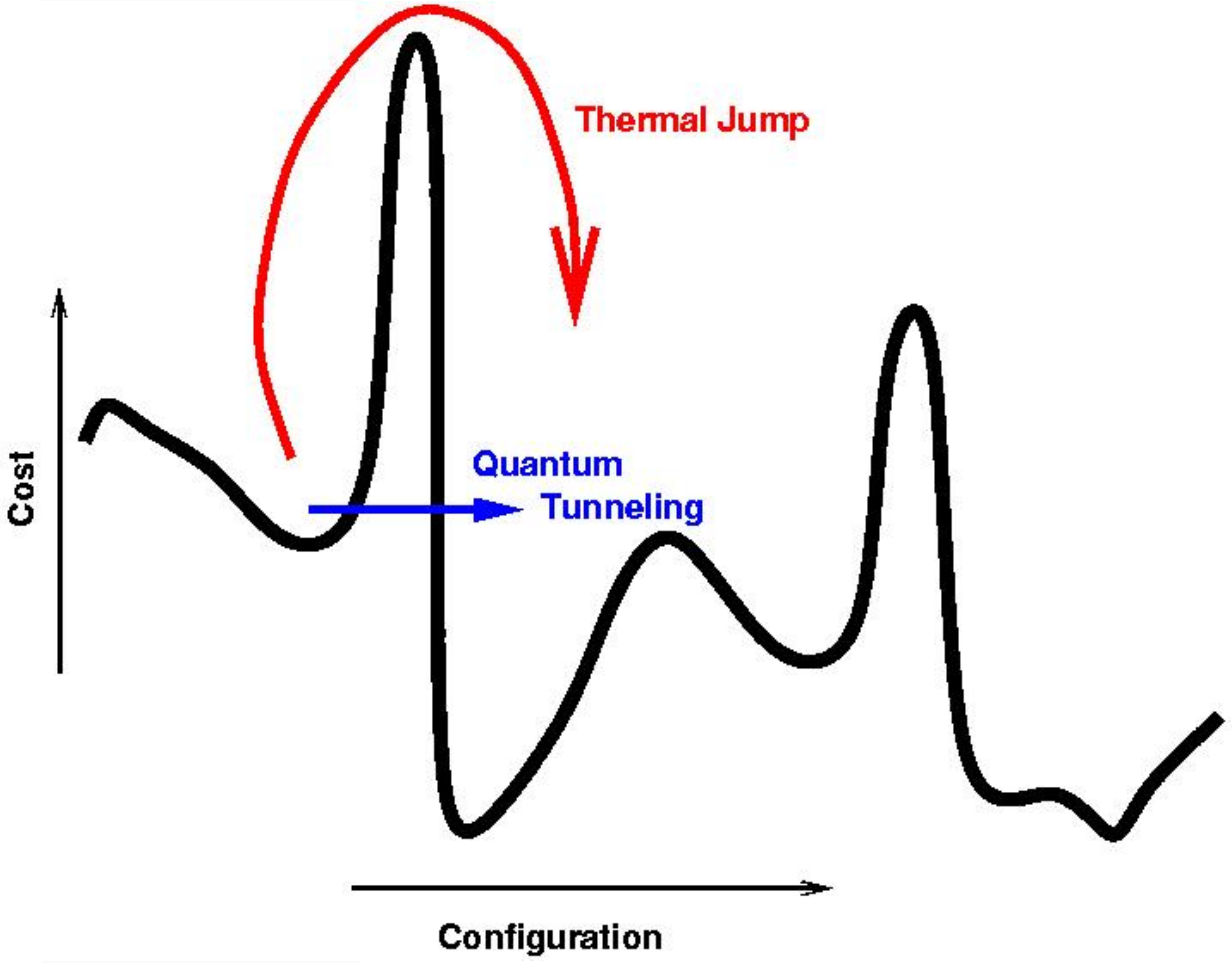
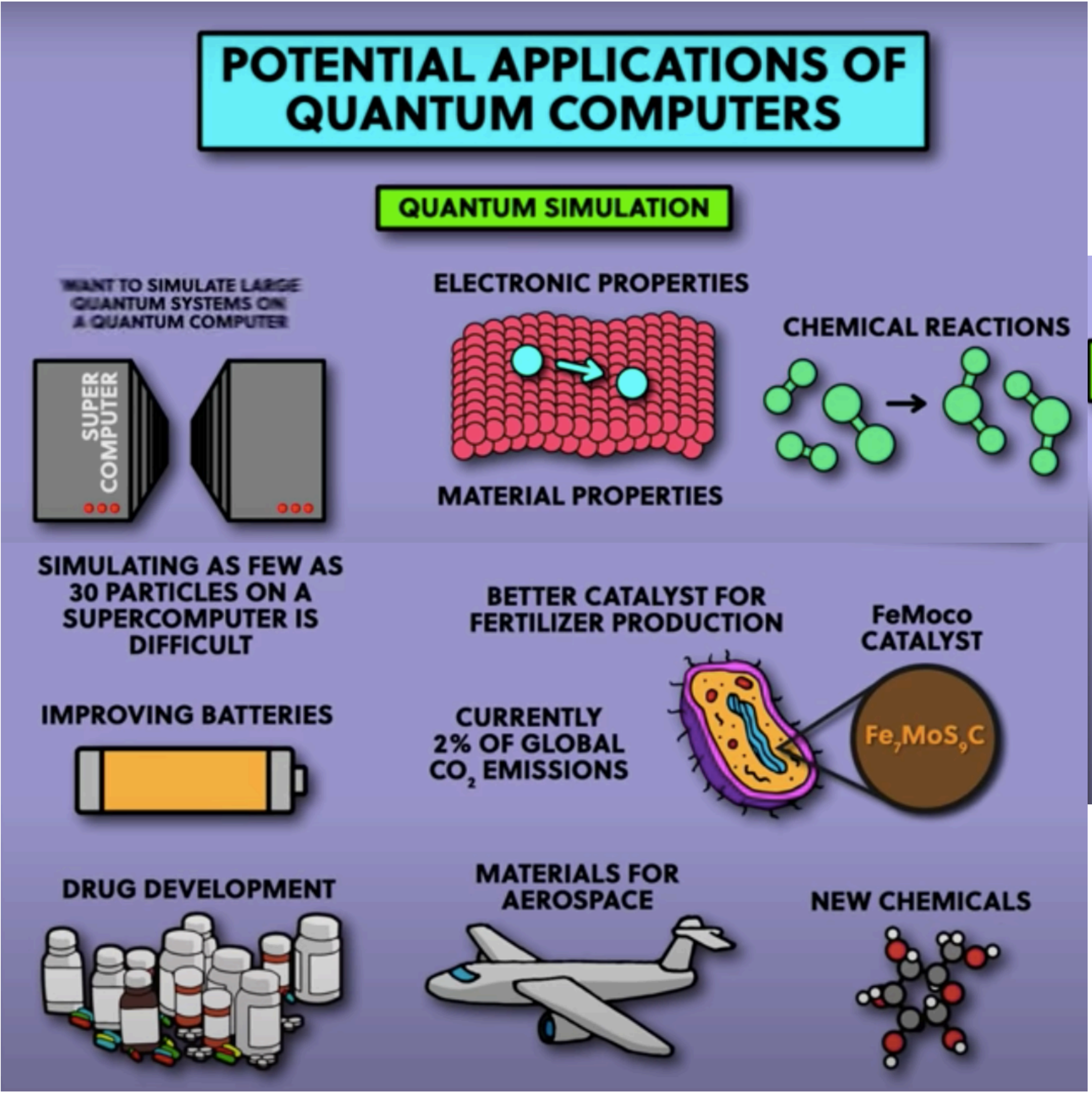
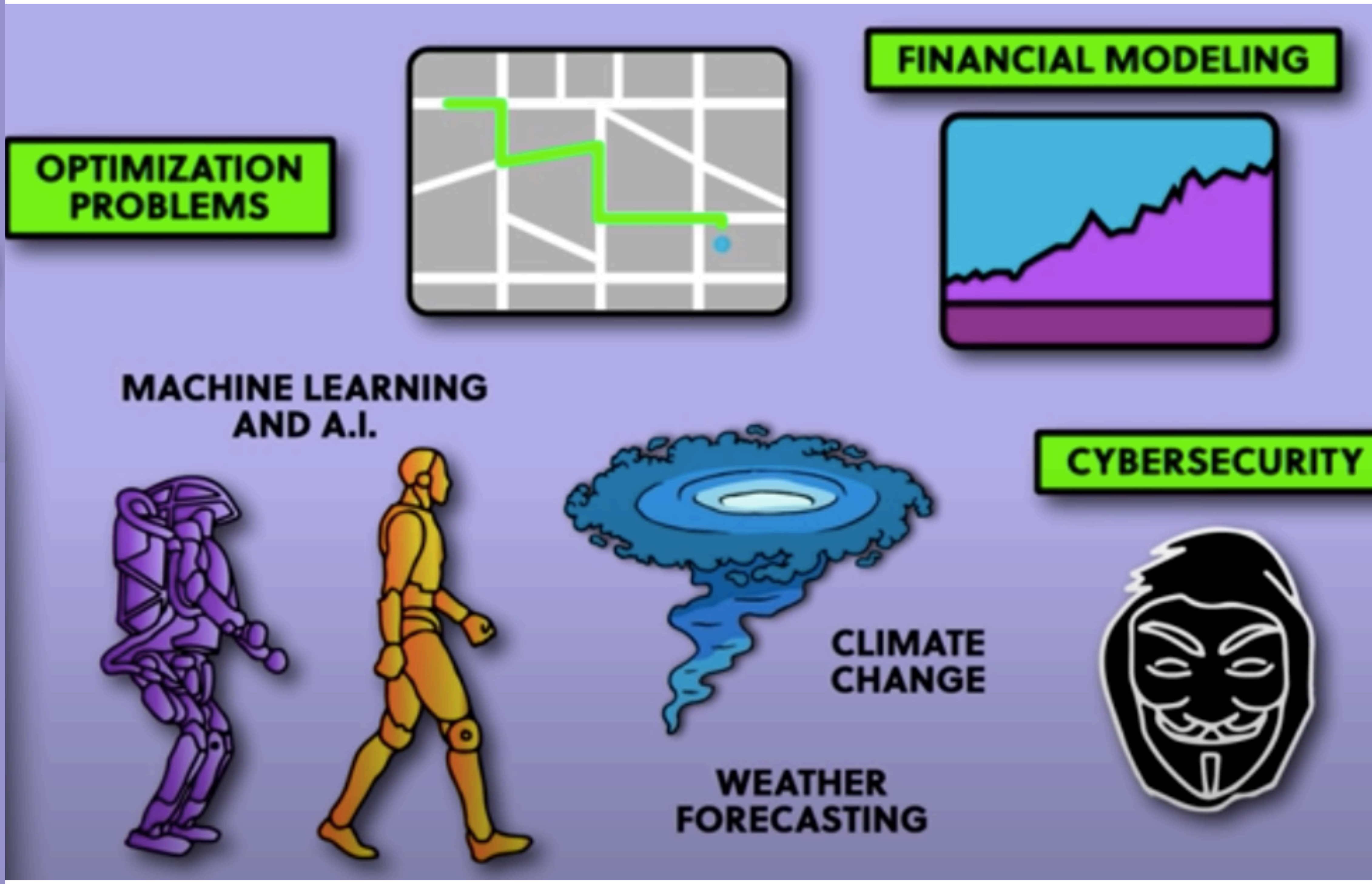




*Fig. 8 Annealing functions $A(s)$, $B(s)$. Annealing begins at $s = 0$ with $A(s) \gg B(s)$ and ends at $s = 1$ with $A(s) \ll B(s)$. Data shown are representative of D-Wave 2X systems.*




POTENTIAL APPLICATIONS OF QUANTUM COMPUTERS
QUANTUM SIMULATION
WANT TO SIMULATE LARGE QUANTUM SYSTEMS ON A QUANTUM COMPUTER
SUPER COMPUTER
ELECTRONIC PROPERTIES
MATERIAL PROPERTIES
CHEMICAL REACTIONS
SIMULATING AS FEW AS 30 PARTICLES ON A SUPERCOMPUTER IS DIFFICULT
BETTER CATALYST FOR FERTILIZER PRODUCTION
FeMoco CATALYST
IMPROVING BATTERIES
CURRENTLY 2% OF GLOBAL $CO_2$ EMISSIONS
$Fe_7MoS_9C$
DRUG DEVELOPMENT
MATERIALS FOR AEROSPACE
NEW CHEMICALS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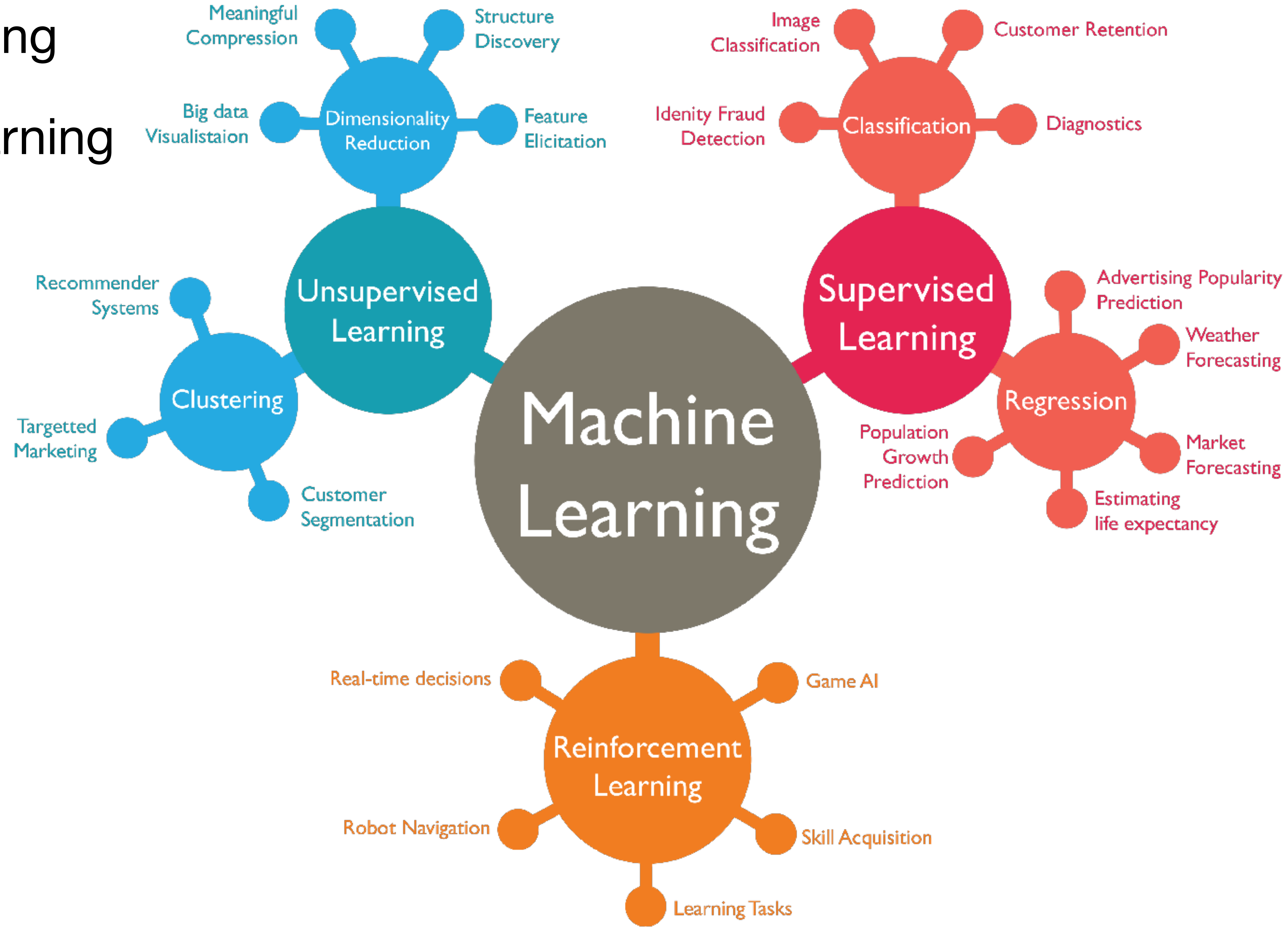
FINANCIAL MODELING
MACHINE LEARNING AND A.I.
CYBERSECURITY
CLIMATE CHANGE
WEATHER FORECASTING

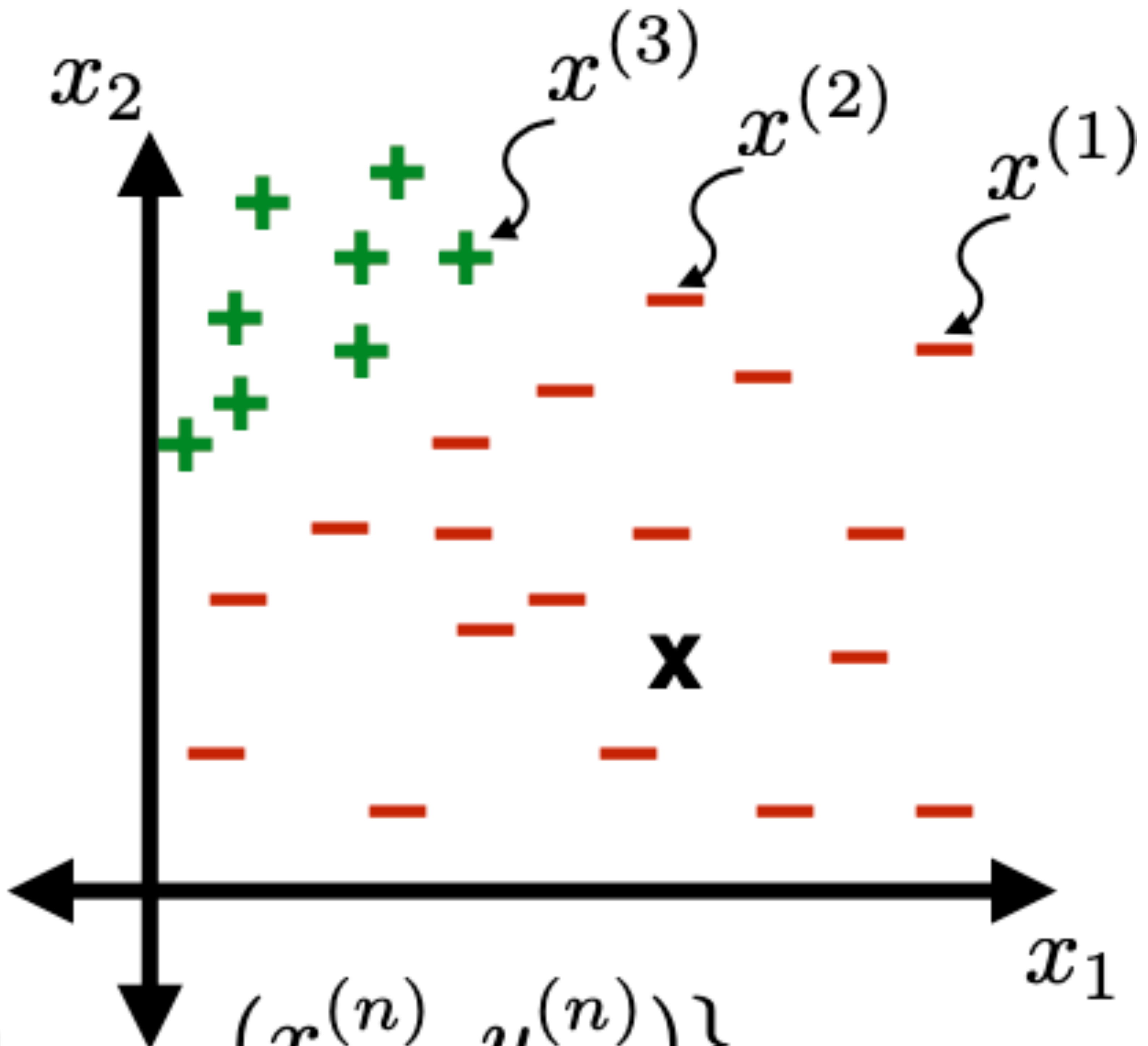
https://quantum-computing.ibm.com/

# Machine Learning

- Supervised machine learning
- Unsupervised machine learning
- Reinforcement learning



# Getting started




**What do we have?** (Training) data

- *n* training data points
- For data point $i \in \{1, \ldots, n\}$
  - Feature vector $x^{(i)} = (x_1^{(i)}, \ldots, x_d^{(i)})^\top \in \mathbb{R}^d$
  - Label $y^{(i)} \in \{-1, +1\}$
- Training data $\mathcal{D}_n = \{(x^{(1)}, y^{(1)}), \ldots, (x^{(n)}, y^{(n)})\}$

**What do we want?** A good way to label new points

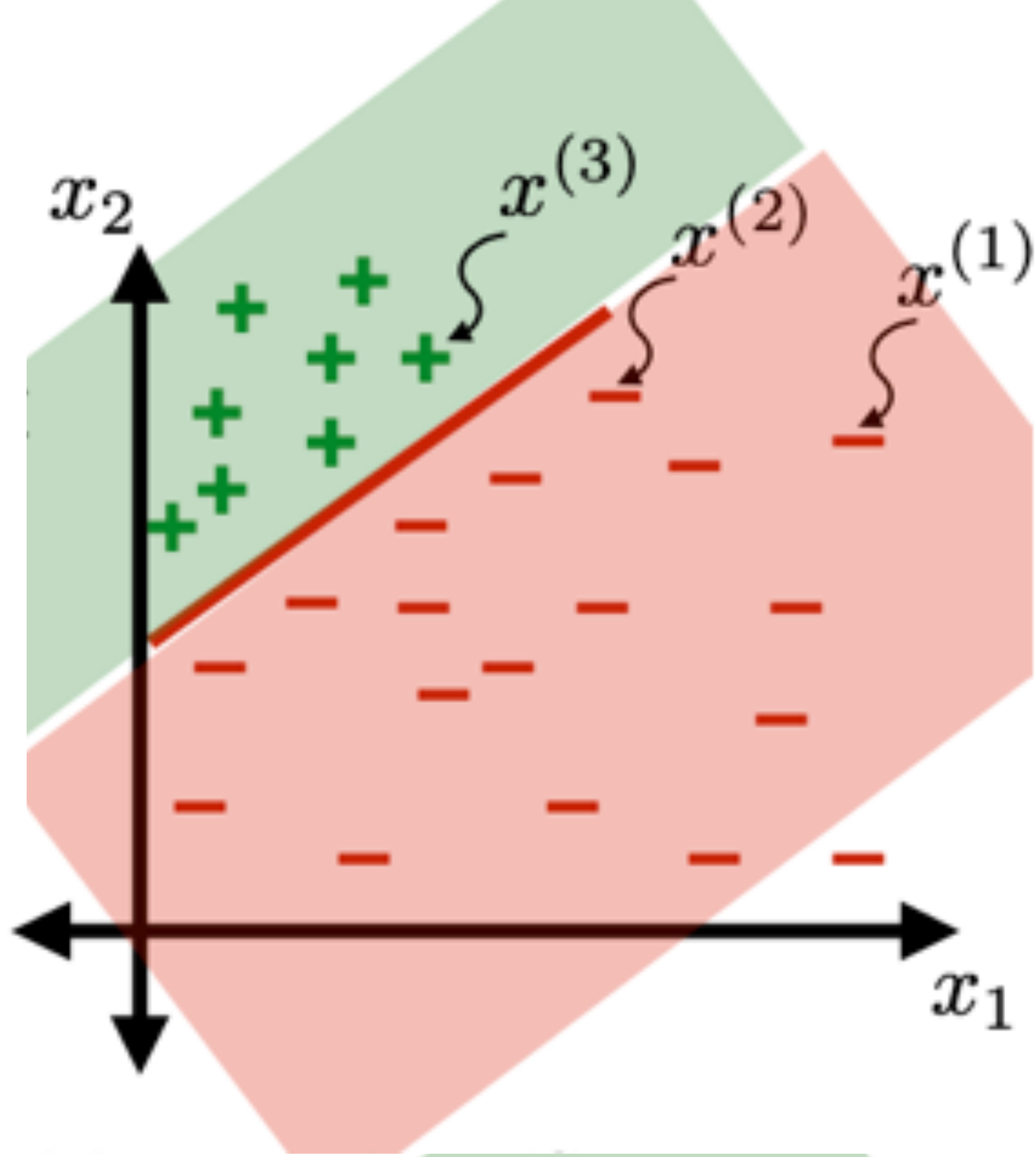
- How to label? Hypothesis $h : \mathbb{R}^d \to \{-1, +1\}$

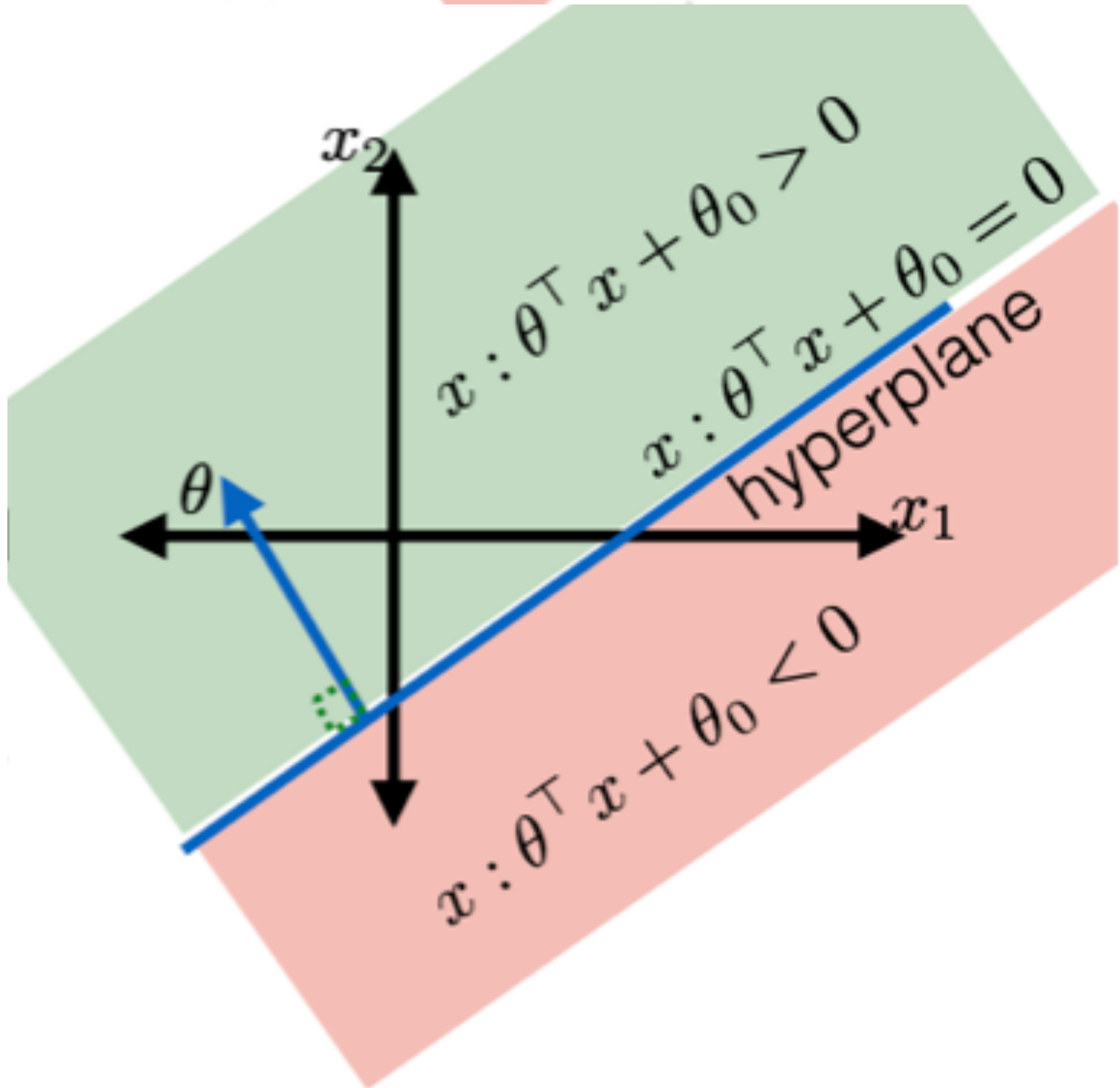

- Example *h*: For any *x*, *h*(*x*) = +1

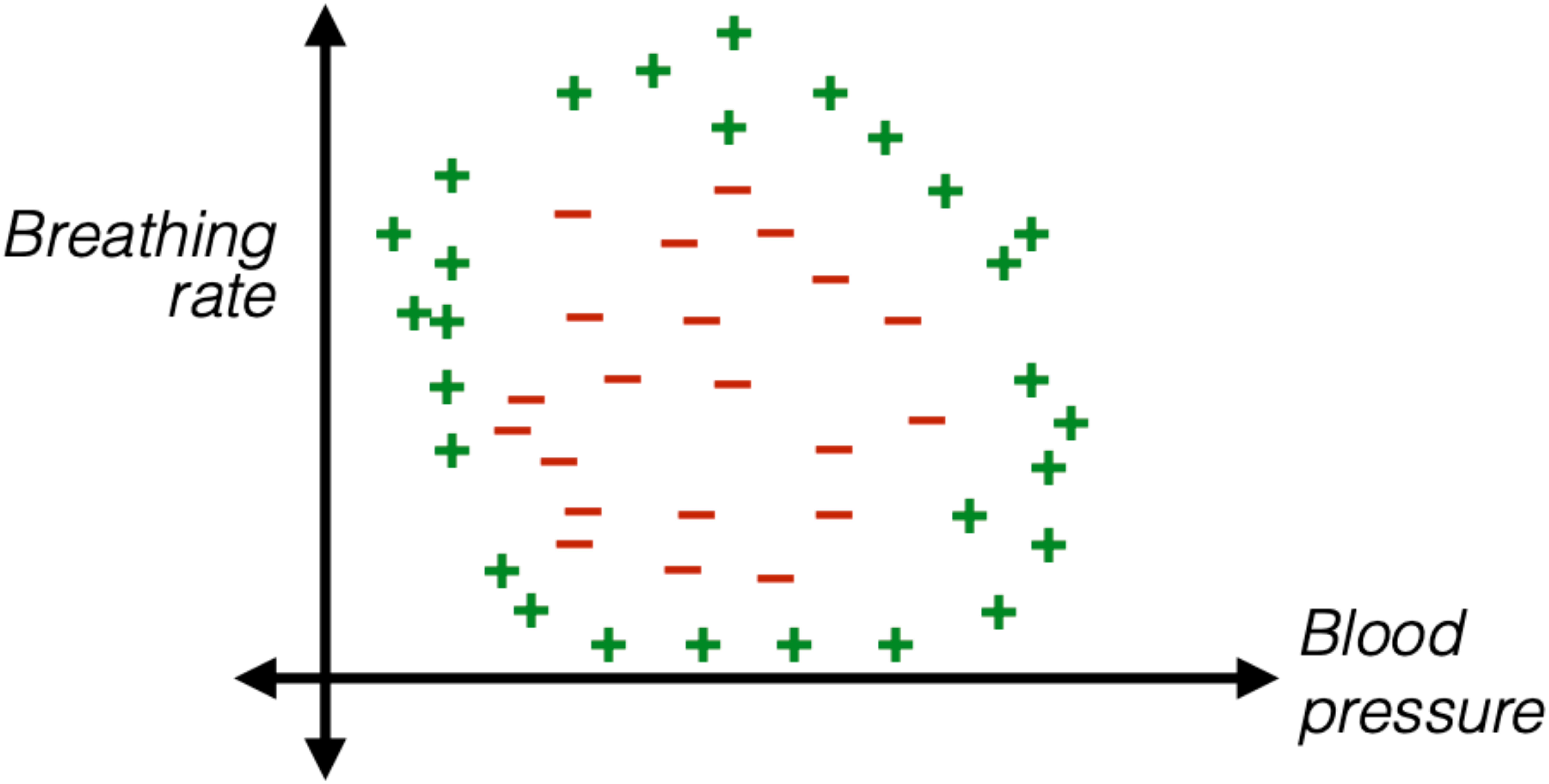
https://tamarabroderick.com/ml.html

# Linear classifiers

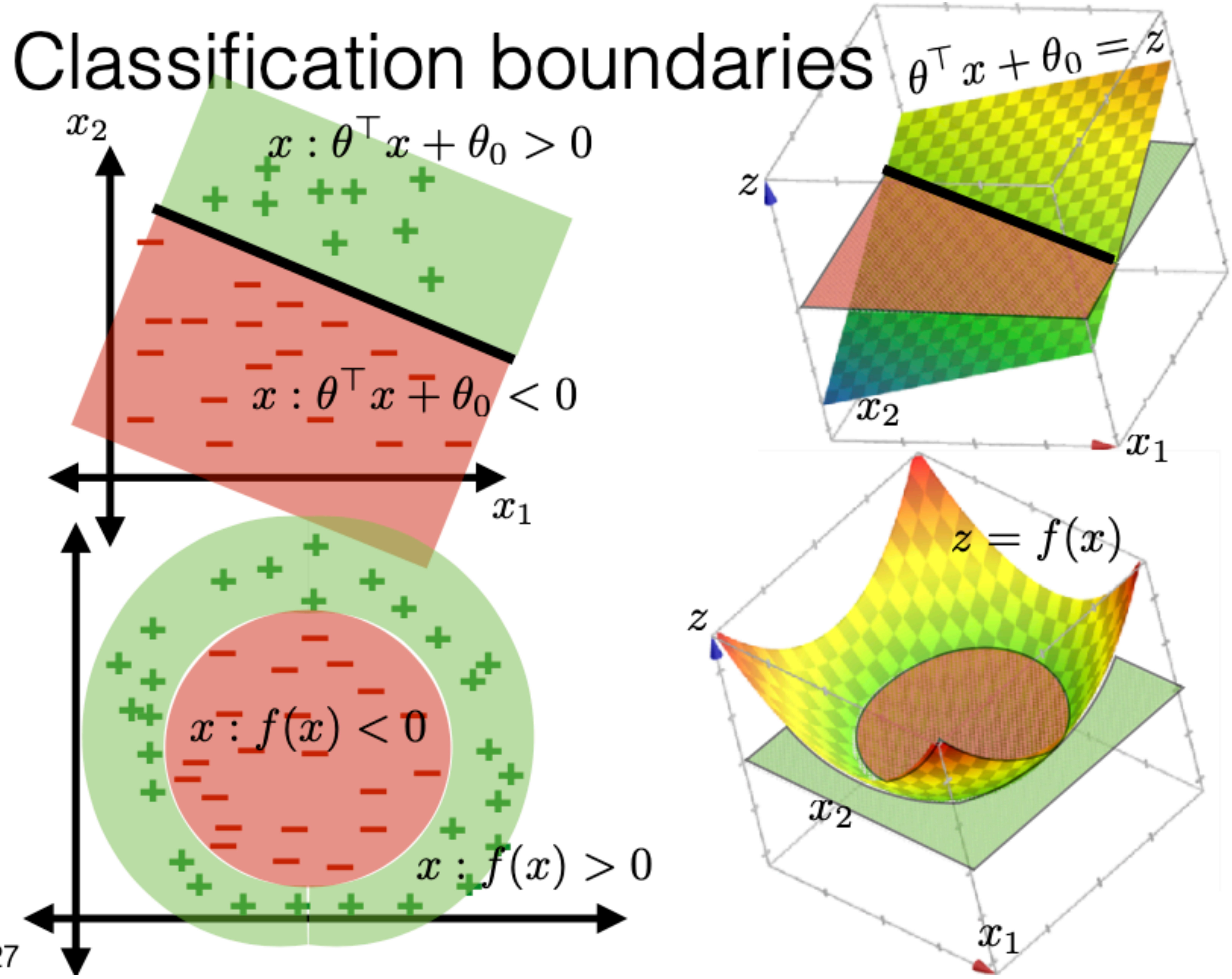
- Hypotesis class $\mathscr{H}$ :  $h \in \mathscr{H}$
- Linear classifier $h(x; \theta, \theta_0) = sign(\theta \cdot x + \theta_0)$
- 0-1 Loss function $L(g, a) = \begin{cases} 0 & if \quad g = a \\ 1 & else \end{cases}$
- training error $\varepsilon_n(h) = \dfrac{1}{n} \sum_{i=1}^{n} L(h(x^{(i)}), y^{(i)})$




Breathing rate
Blood pressure

# Classification boundaries



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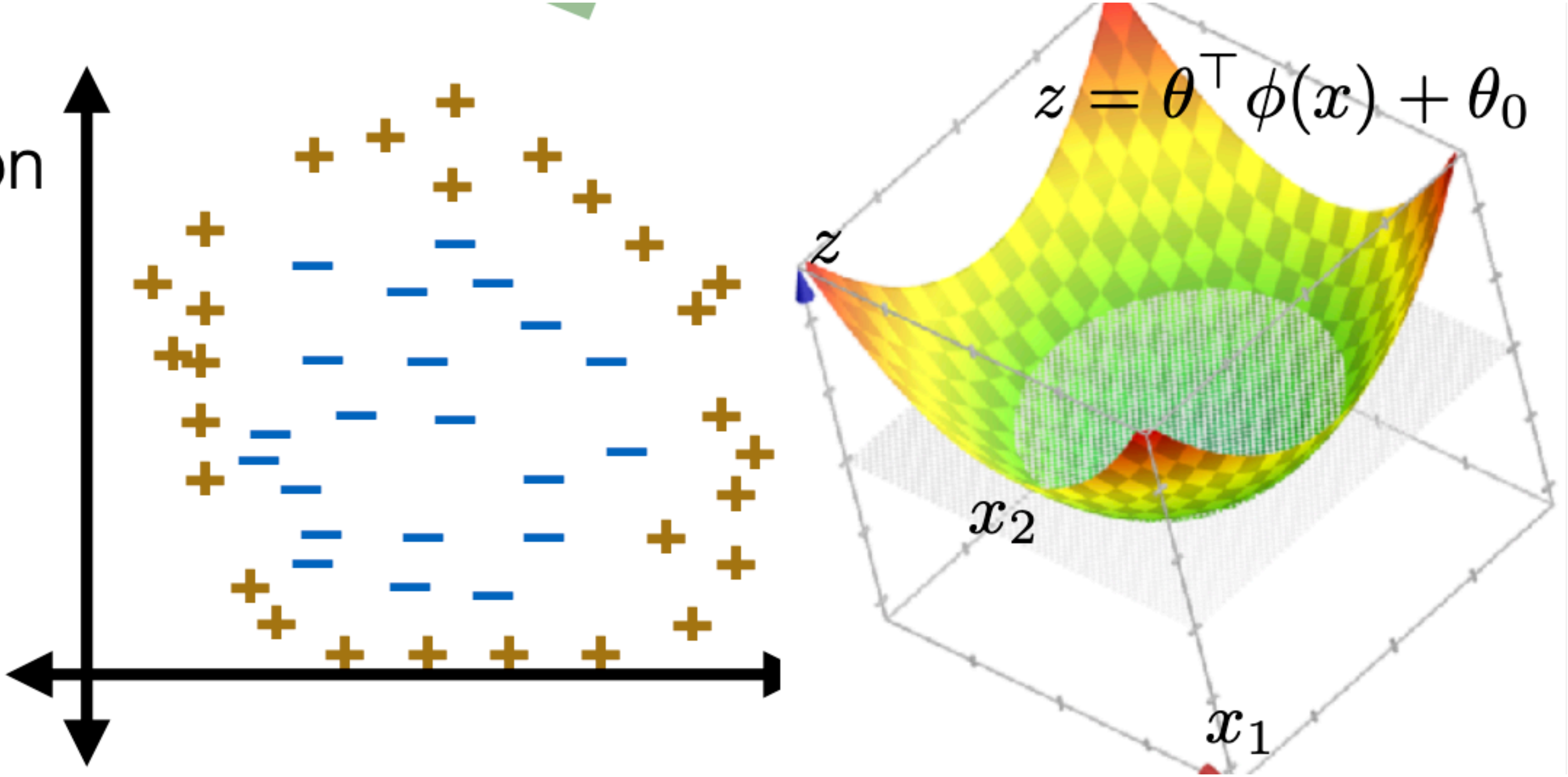
# Linear Classification with Polinomial Features

- Linear classification with polynomial features:

$\phi(x) = [x_1, x_2, x_1^2, x_1 x_2, x_2^2]^\top$

1

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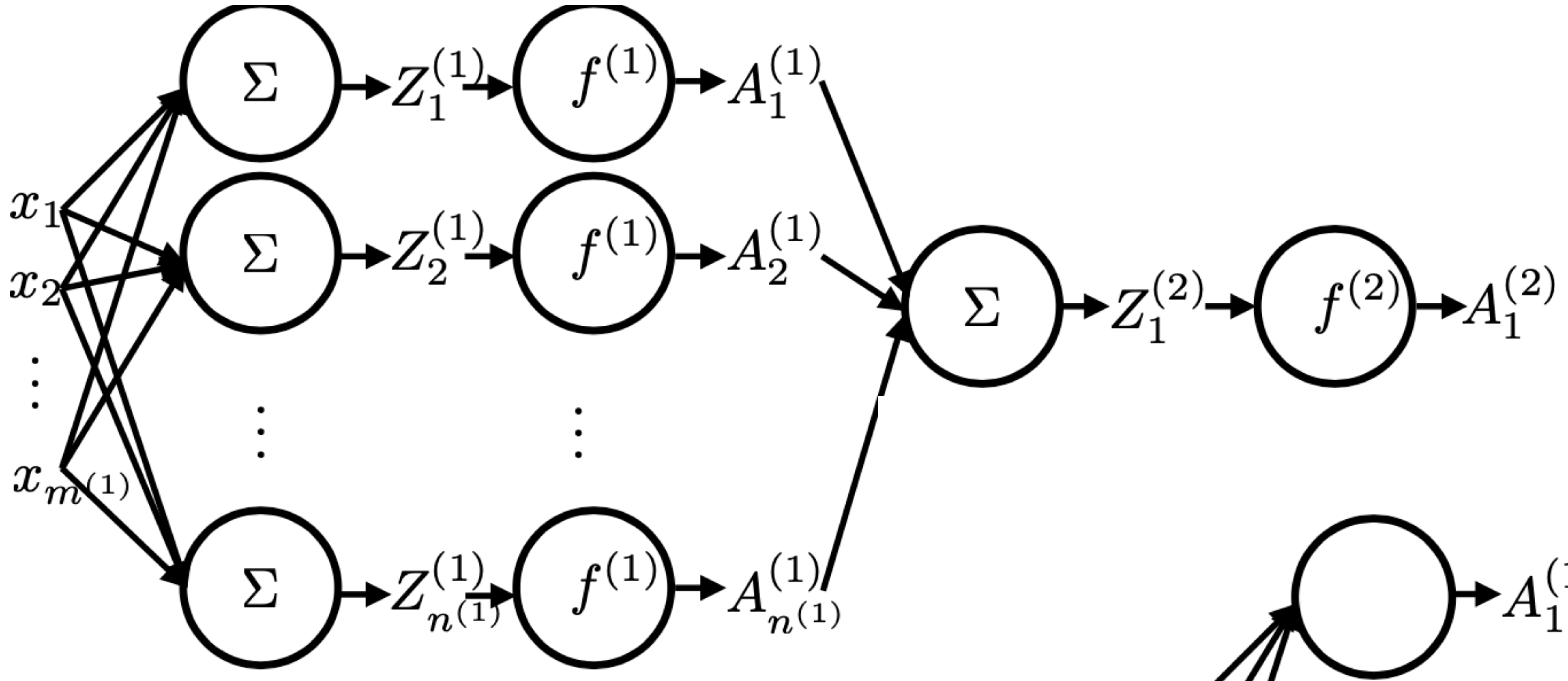
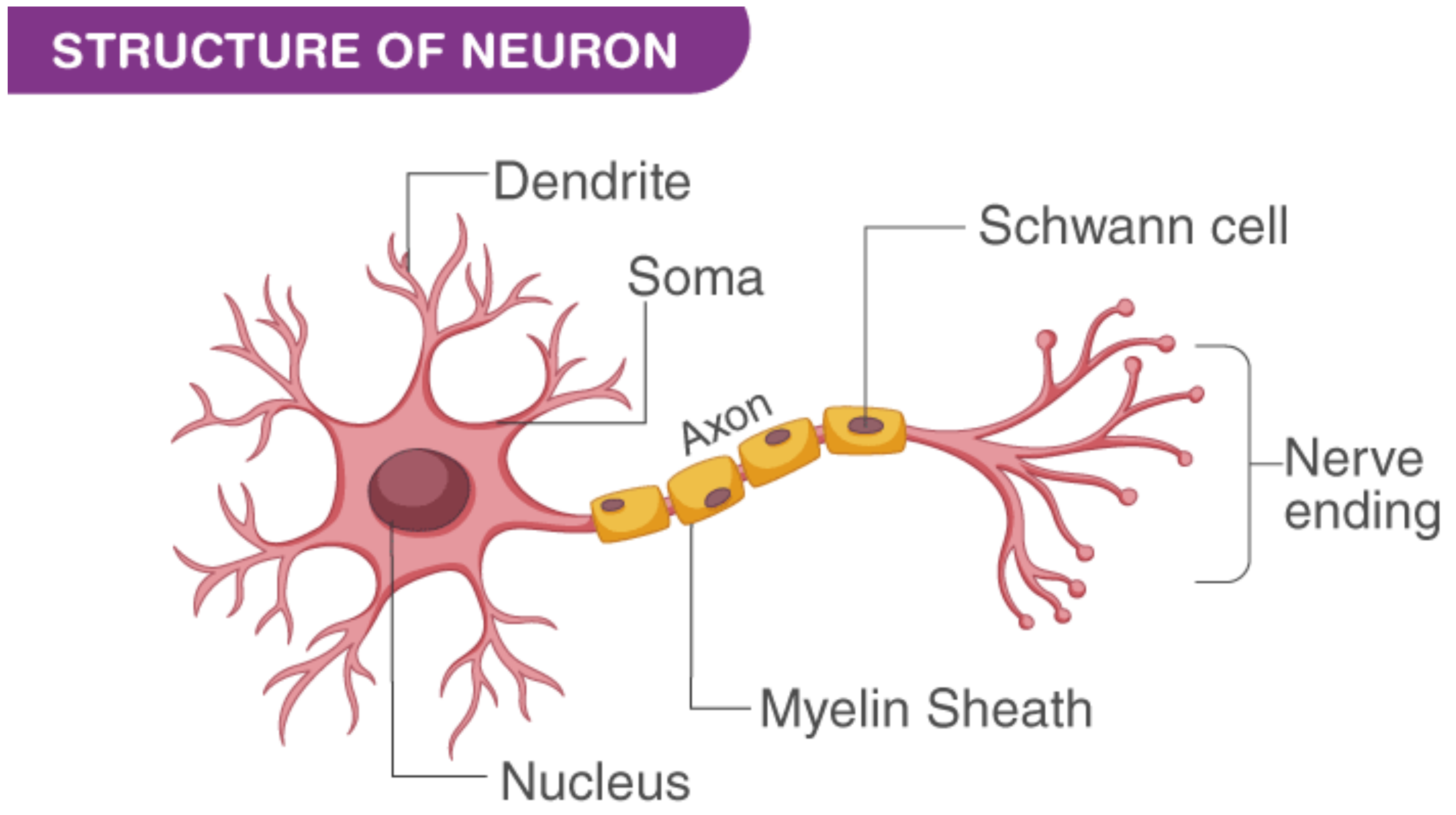
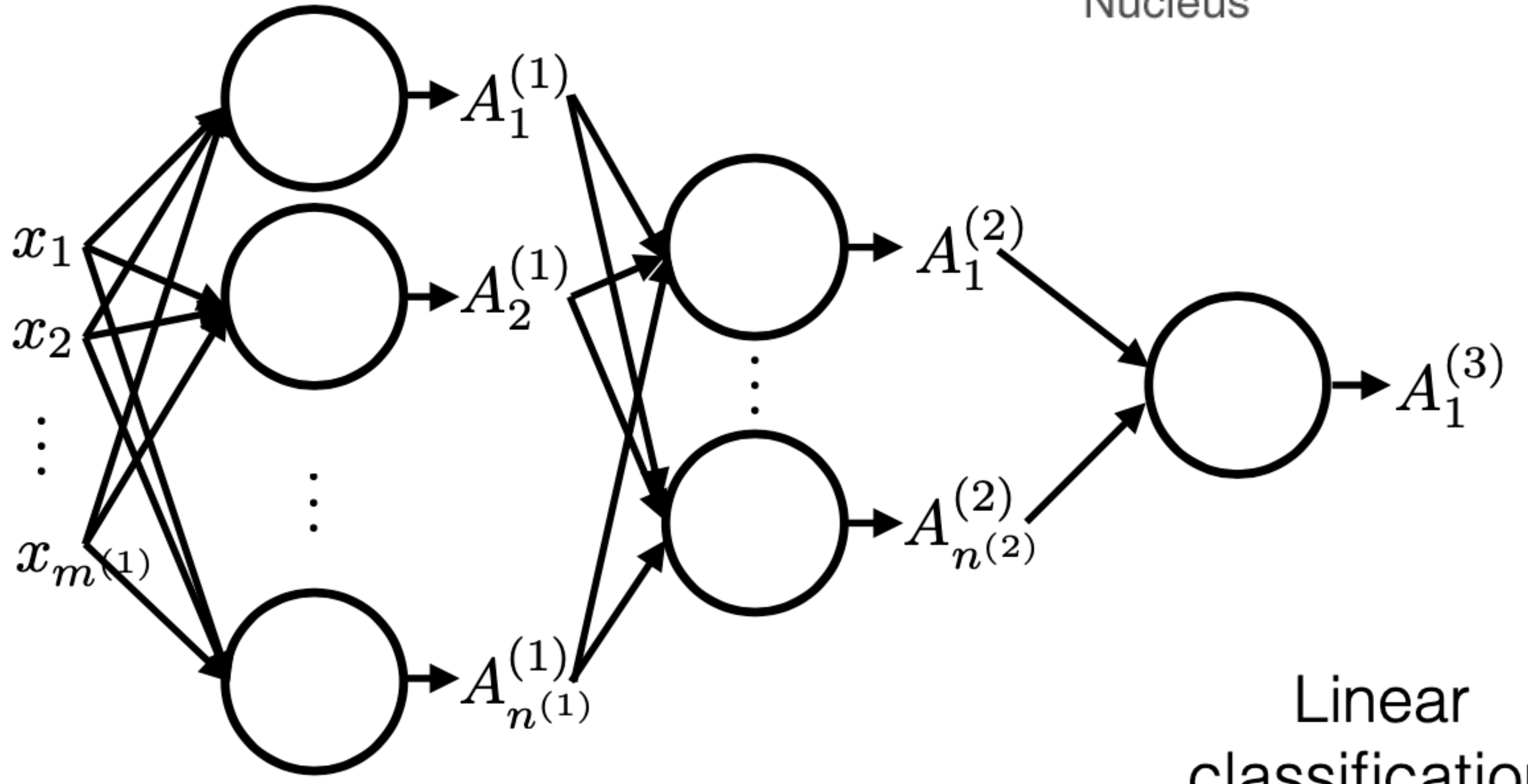
# Neural Network



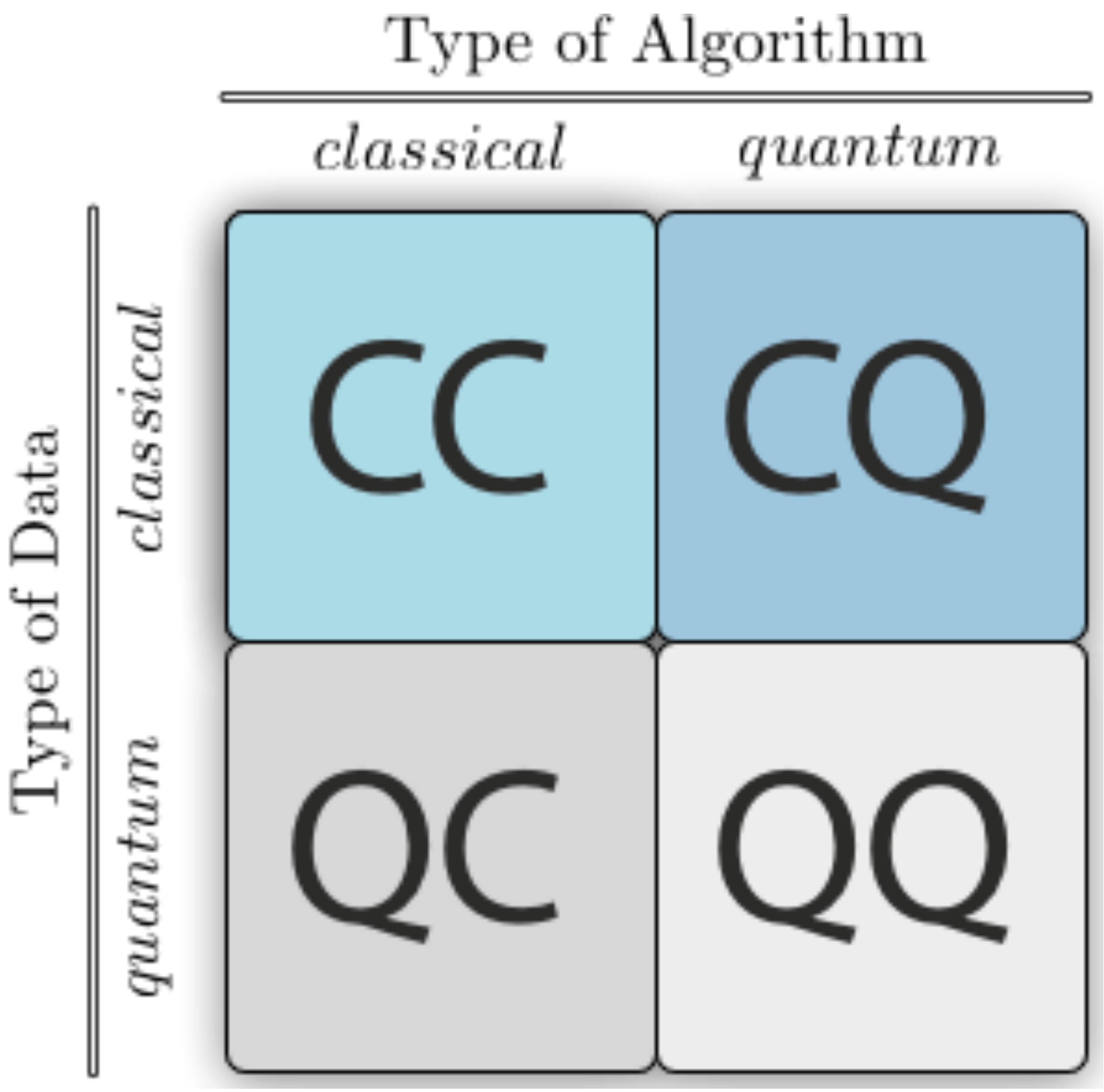



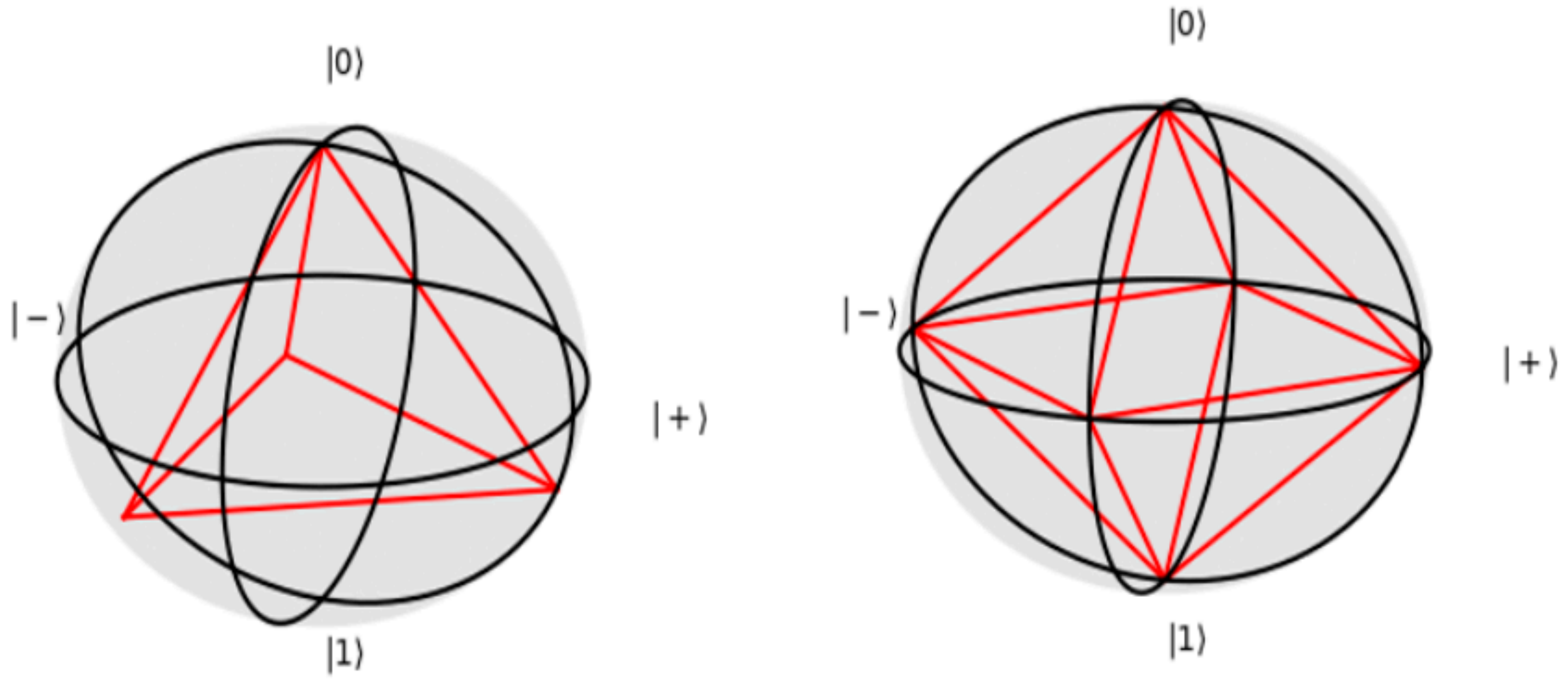



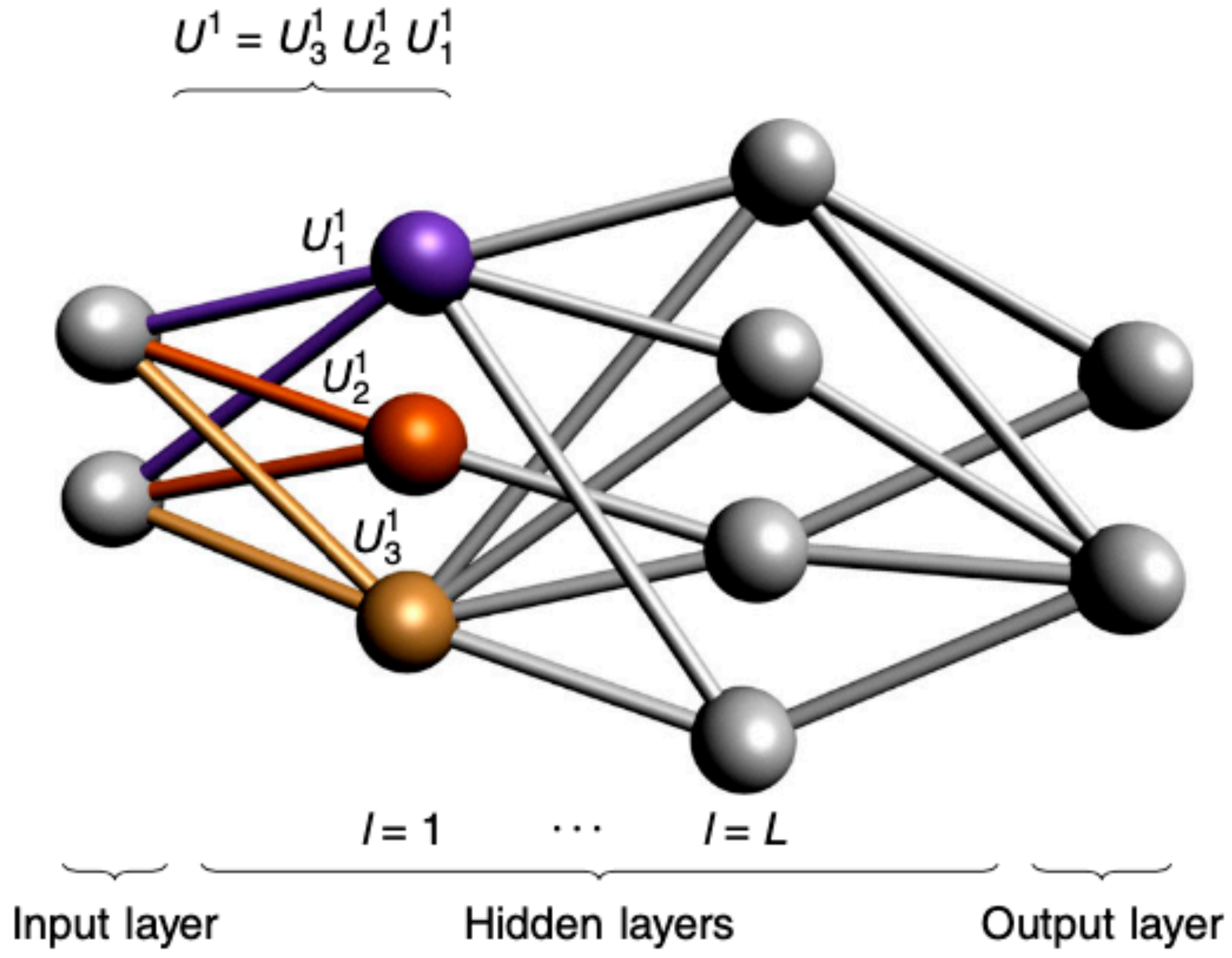

Making features

Linear classification/ regression

# Quantum machine learning




General quantum feedforward neural network https://learn.qiskit.org/course/machine-learning/introduction