PERCOLATION

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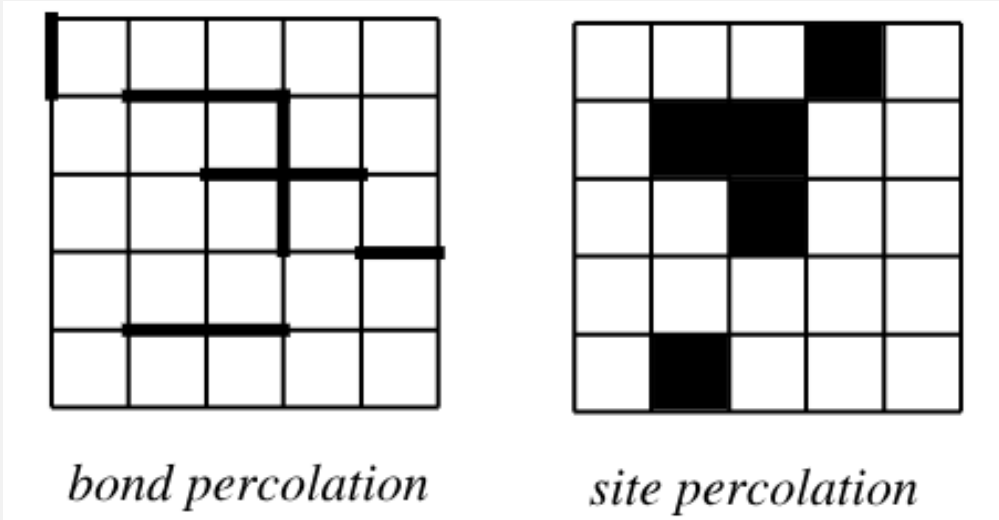
WHAT IS PERCOLATION?

Percolation theory in statistical physics describes the behavior of a network when nodes or links are added with a definite probability and we want to explore how a signal spreads through the network. We will explore phase transition in this model in what follows (In particular, site percolation in a 2D lattice).

MODELS OF PERCOLATION



EXAMPLES OF PERCOLATION:

- Fire
- Oil
- Coffee
- Spread of diseases
- Diffusion of information in social media
- ….

PERCOLATION, A BRIEF STUDY

- A question that is studied in percolation is whether there exists a cluster of connected sites of infinite size in an infinite network.

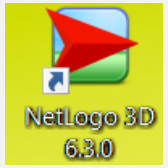
- In percolation theory we can prove that this probability is either 0 or 1 and that the probability is an increasing function of density. This suggests there must be a critical density below which the probability is 0 and above which the probability is 1. Though when we don't take the limit of the size of the network to infinity, we have a continuous phase transition.

PERCOLATION, A BRIEF STUDY

- Critical exponents describe the behavior of physical quantities near continuous phase transitions. It can be shown that percolation clusters become a fractal at values of density sufficiently close to the critical density. It can also be shown that the dimension of the fractal is $M \cong L^{df}$ For large network sizes where M is the size of the giant cluster. It is also shown that the dimension of the fractal is less that 2 when density is less that the critical density and equal to 2 otherwise.

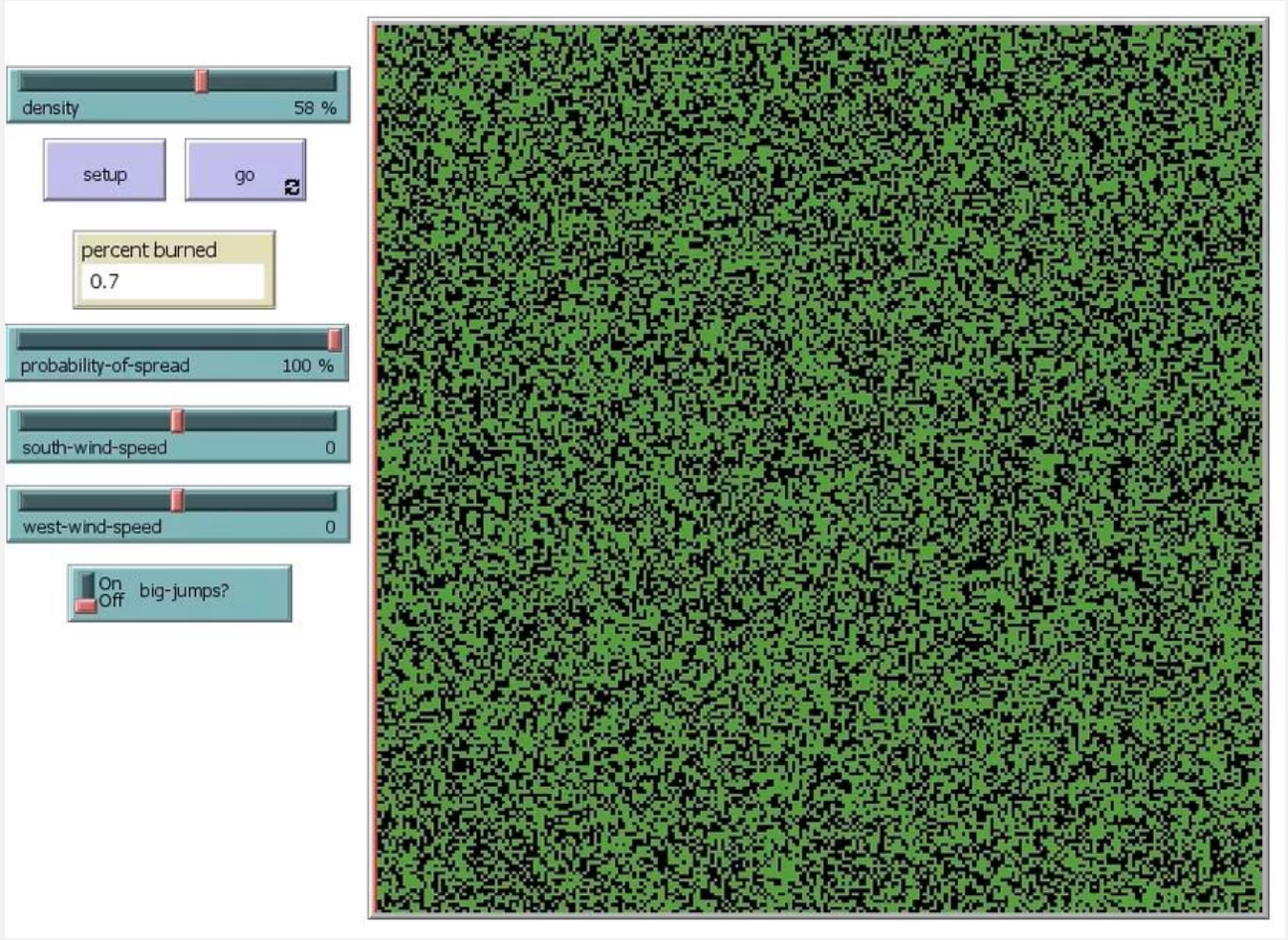
SIMULATION OF PERCOLATION

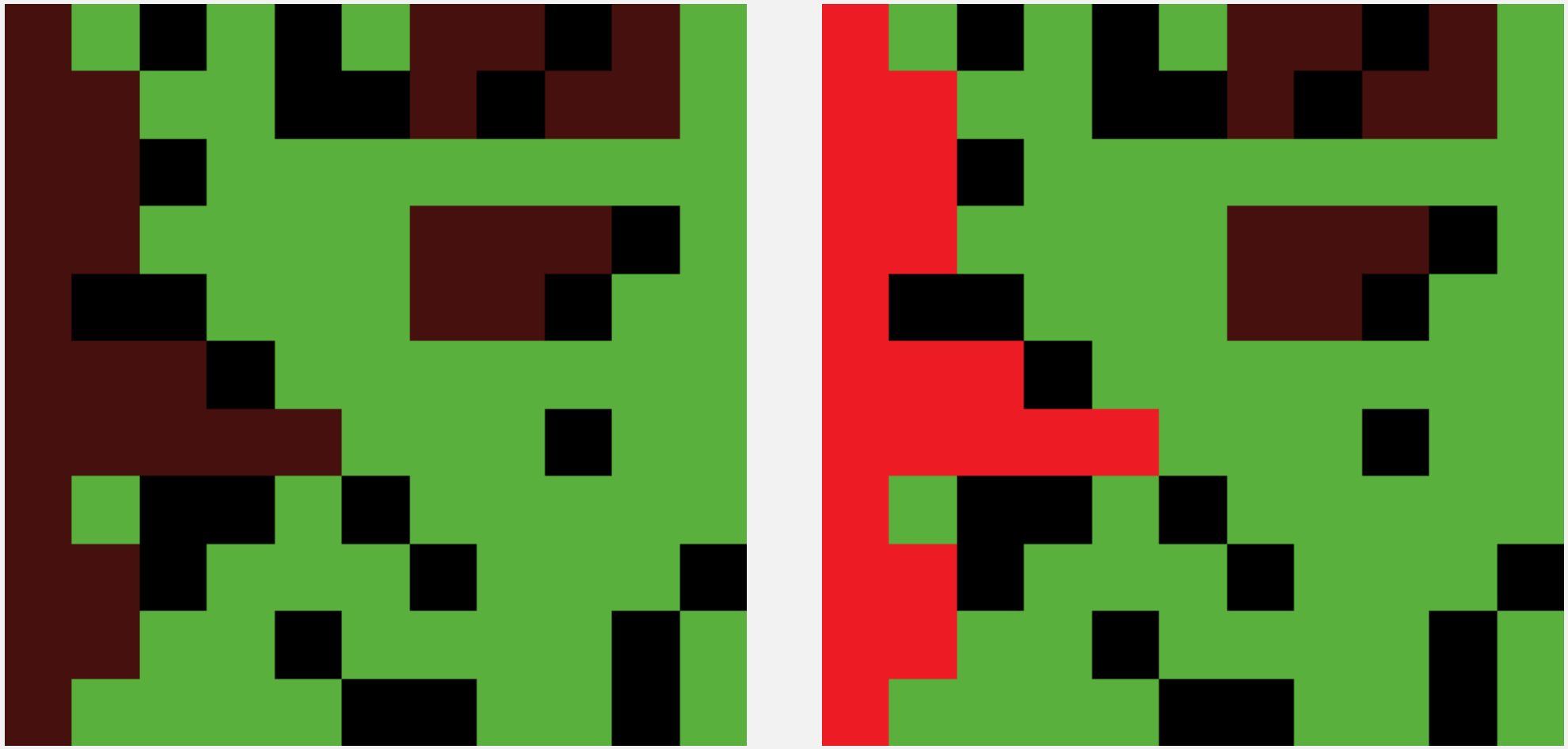
NetLego: NetLogo is an open source, cross-platform tool that enables users to model a wide variety of natural and social phenomena (including biology, chemistry, computer science, economics, physics, psychology, art, and much more).

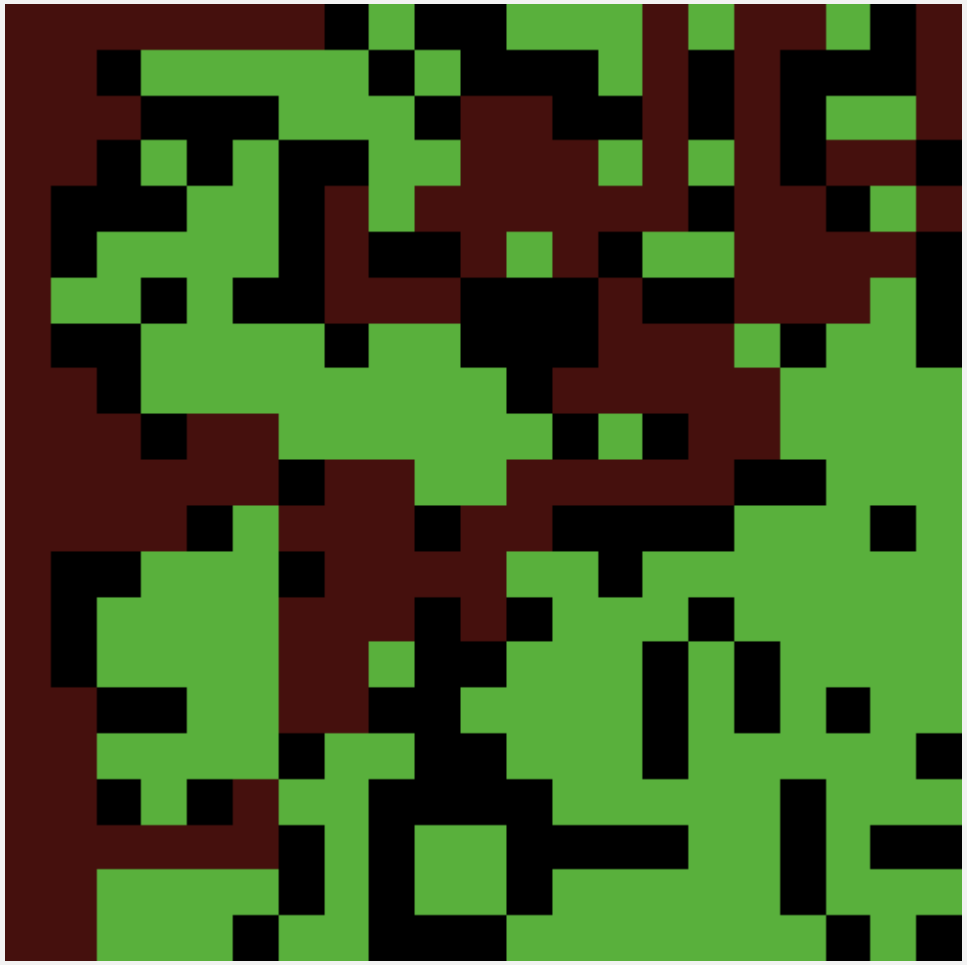


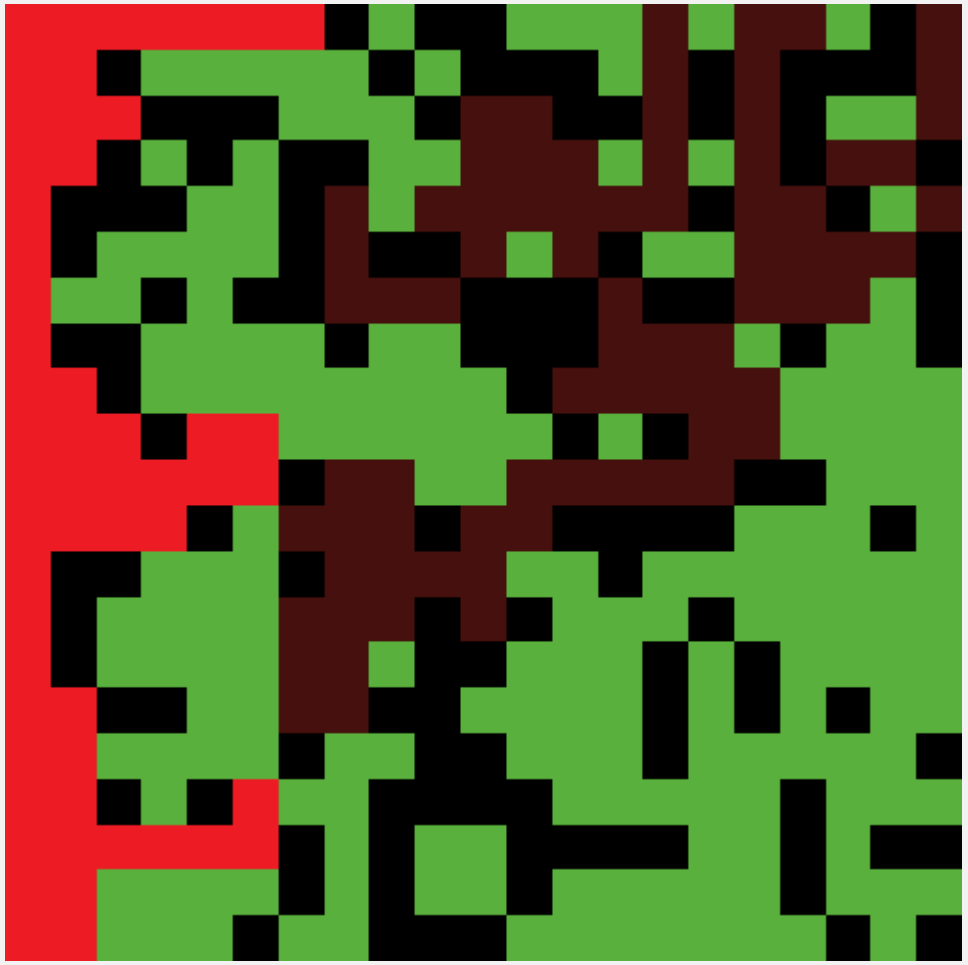
FIRE EXAMPLE

This model simulates the spread of a fire through a forest. It shows that the fire's chance of reaching the right edge of the forest depends critically on the density of trees. This is an example of a common feature of complex systems with some adjustable parameter like probability of spread,west wind speed,south wind speed and Big Jump.





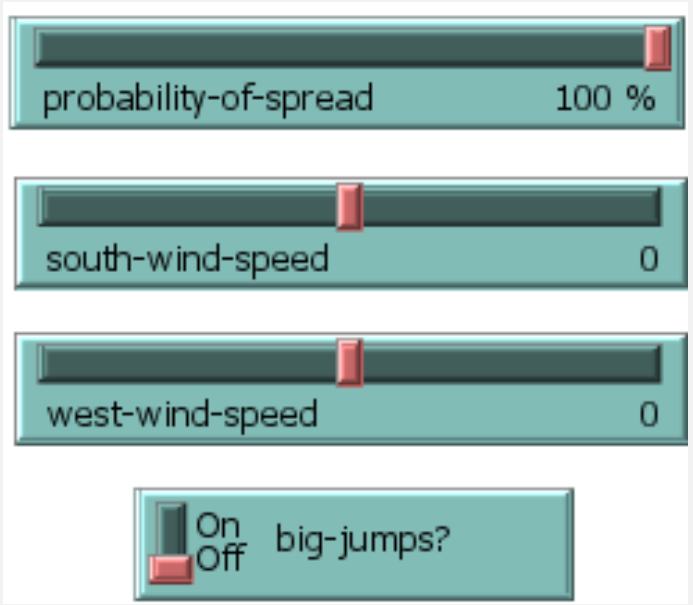




ADJUSTABLE PARAMETERS

- Density
- Size of network
- Probability of spread
- West wind speed
- South wind speed
- Big jump

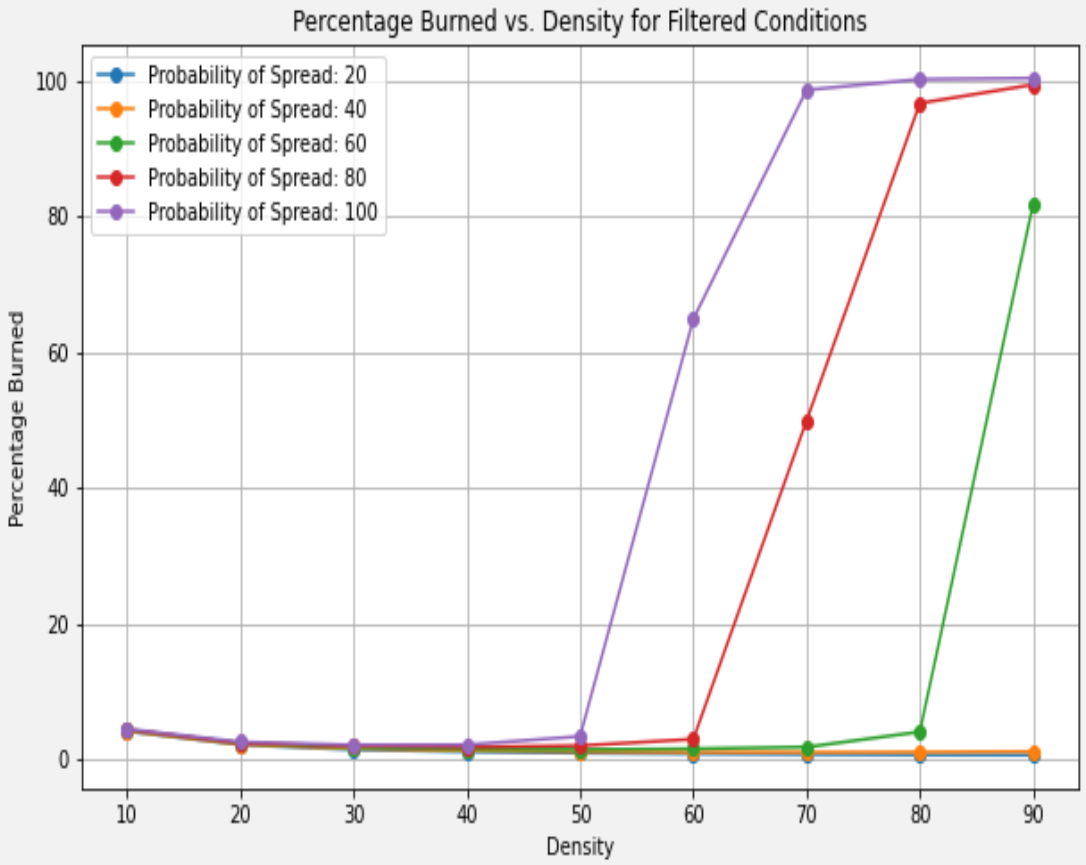
Default mode:

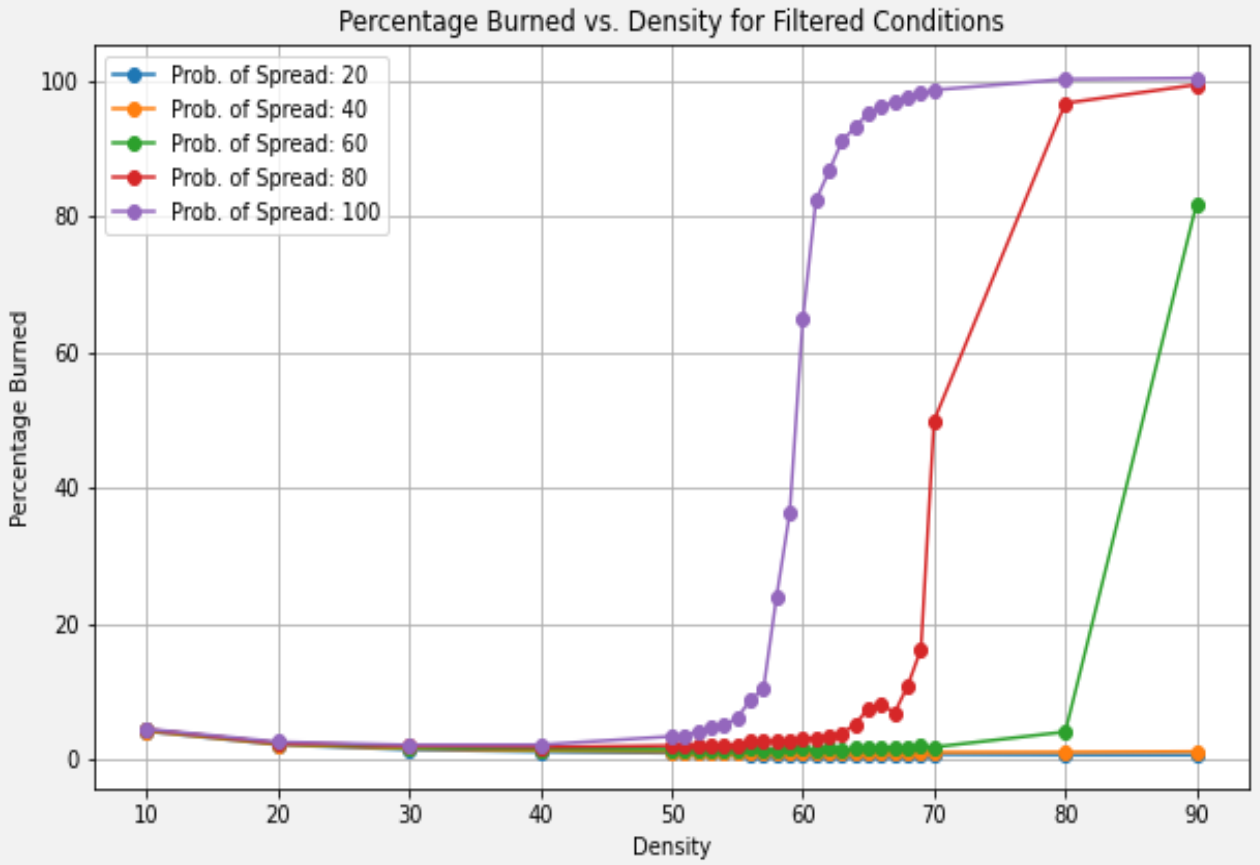


OUR SIMULATION

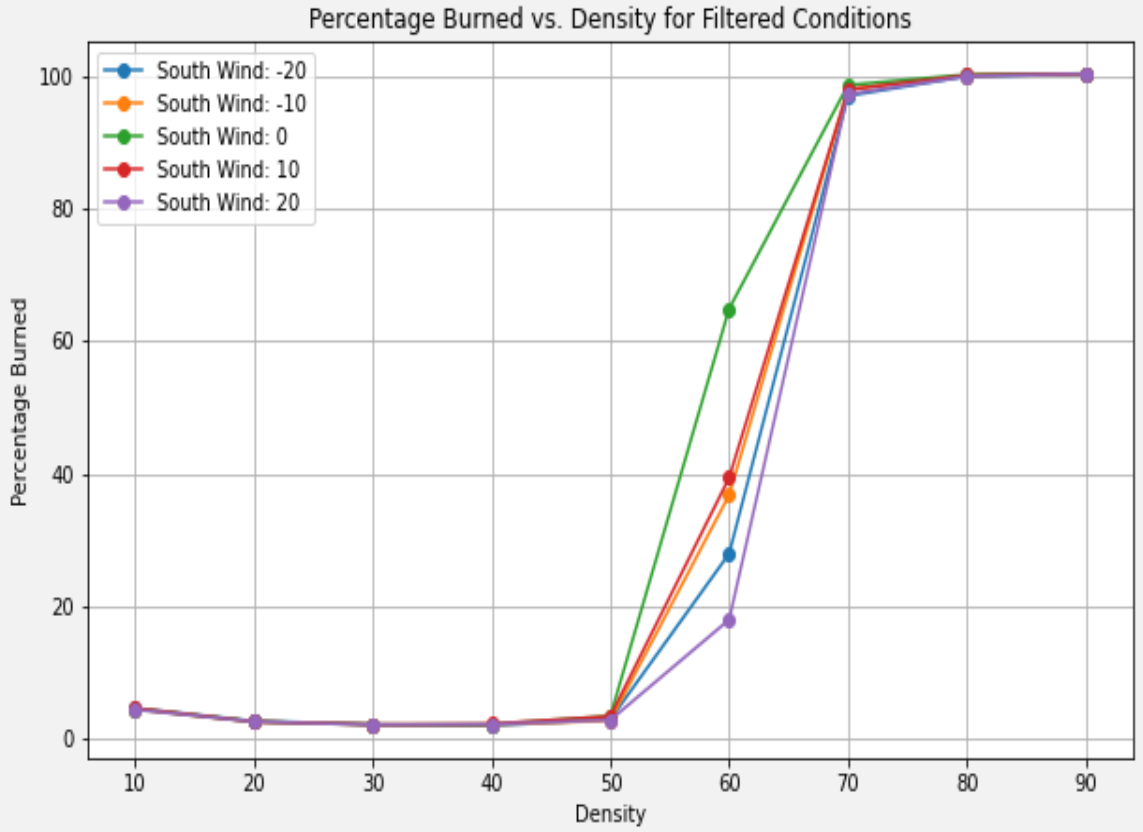
- In this simulation, we have investigated the critical density and how it changes when different features are added to the model. We have also used the model to make an estimation of the dimension of the fractal.

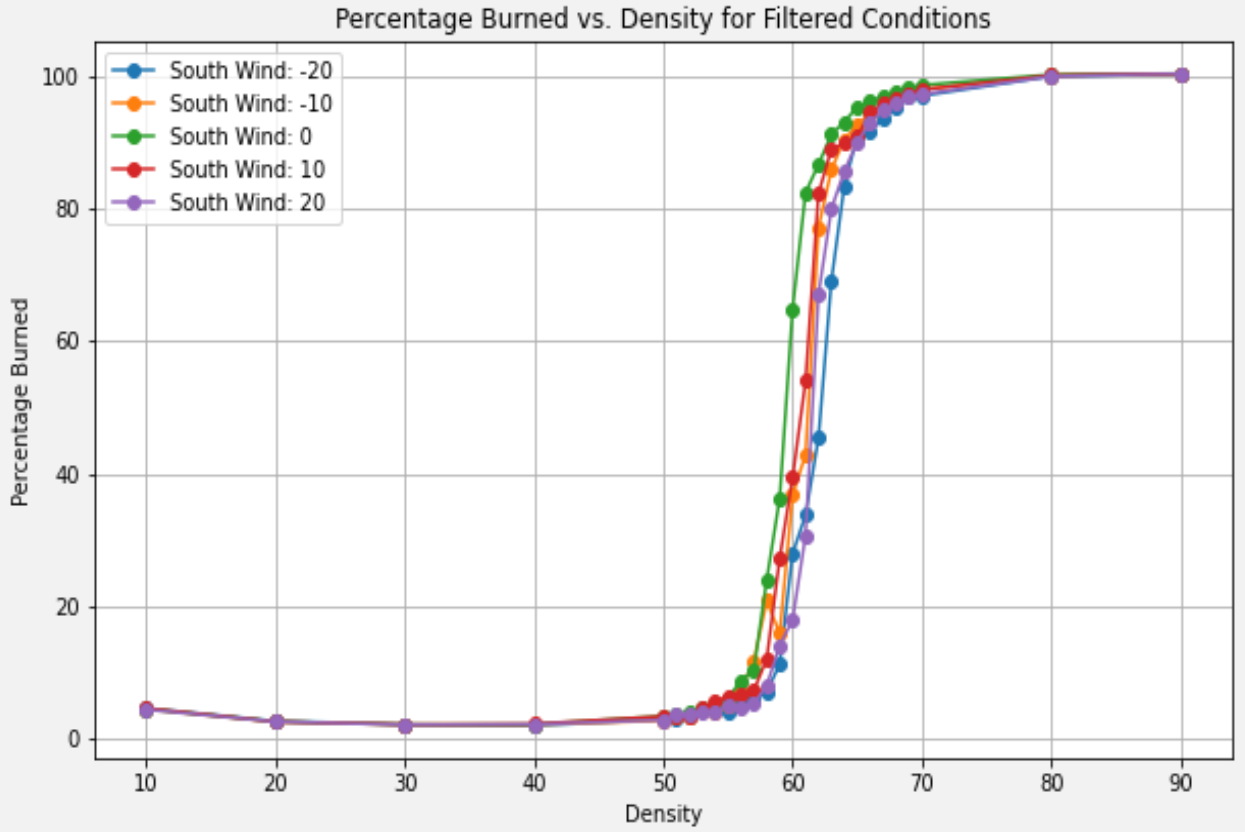
GRAPH OF BURNT PERCENTAGE ACCORDING TO DENSITY WITH GRID OF SIZE 901*901 (BY CHANGING PROBABILITY OF SPREAD)



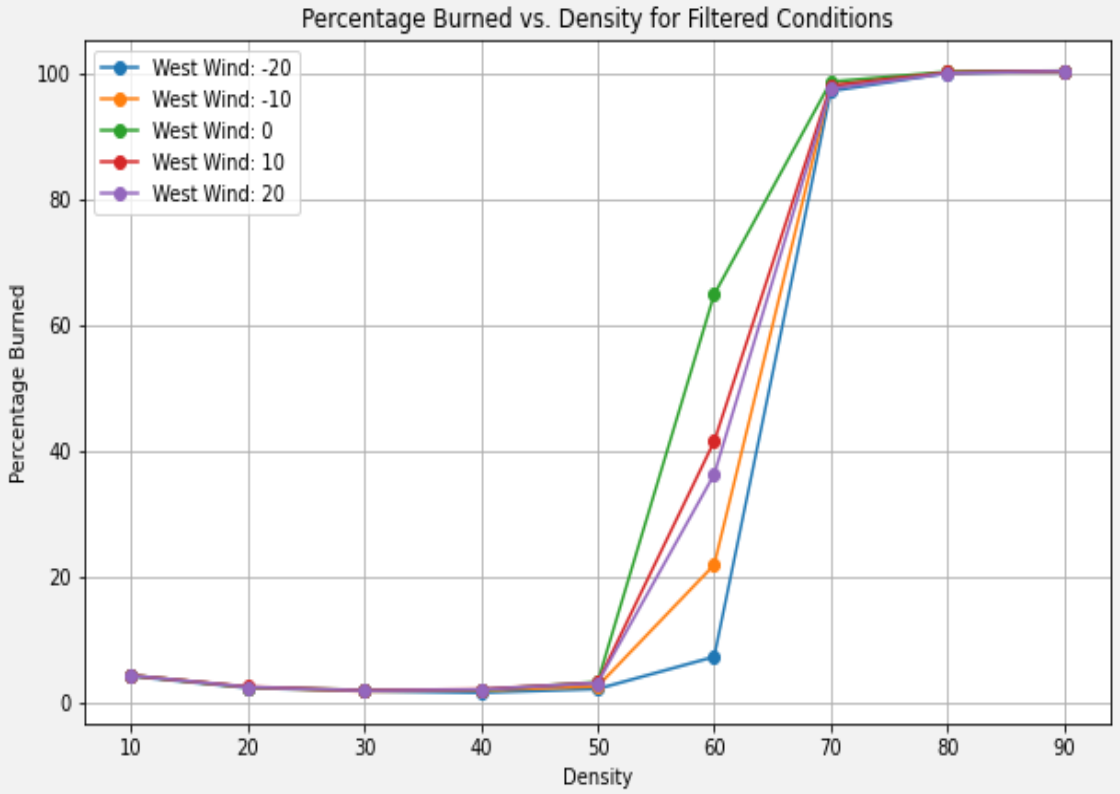


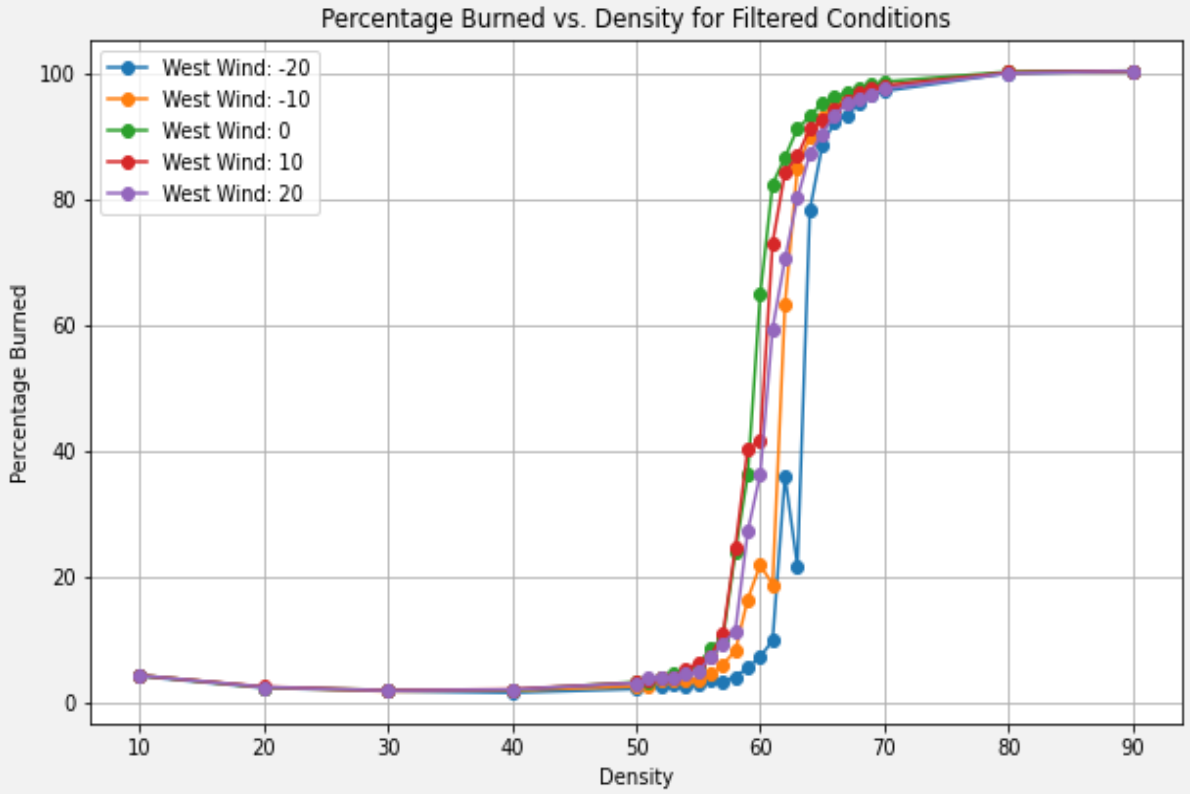
GRAPH OF BURNT PERCENTAGE ACCORDING TO DENSITY WITH GRID OF SIZE 901*901 (BY CHANGING SOUTH-WIND SPEED)



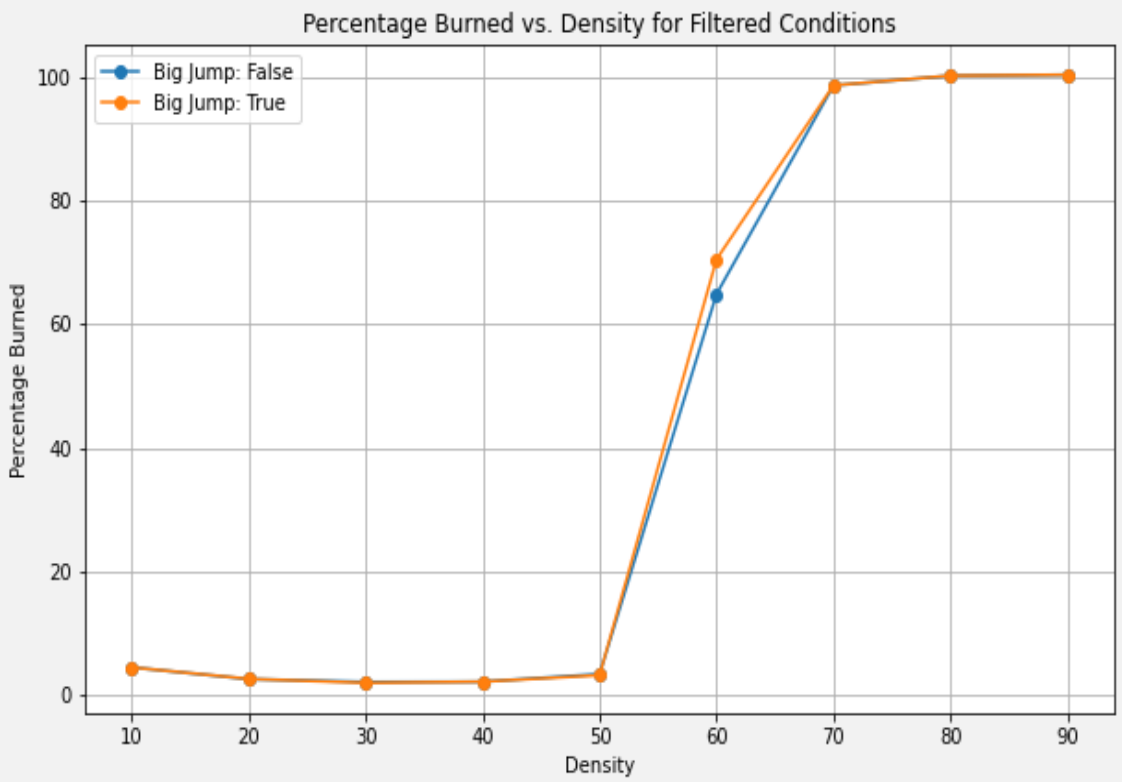


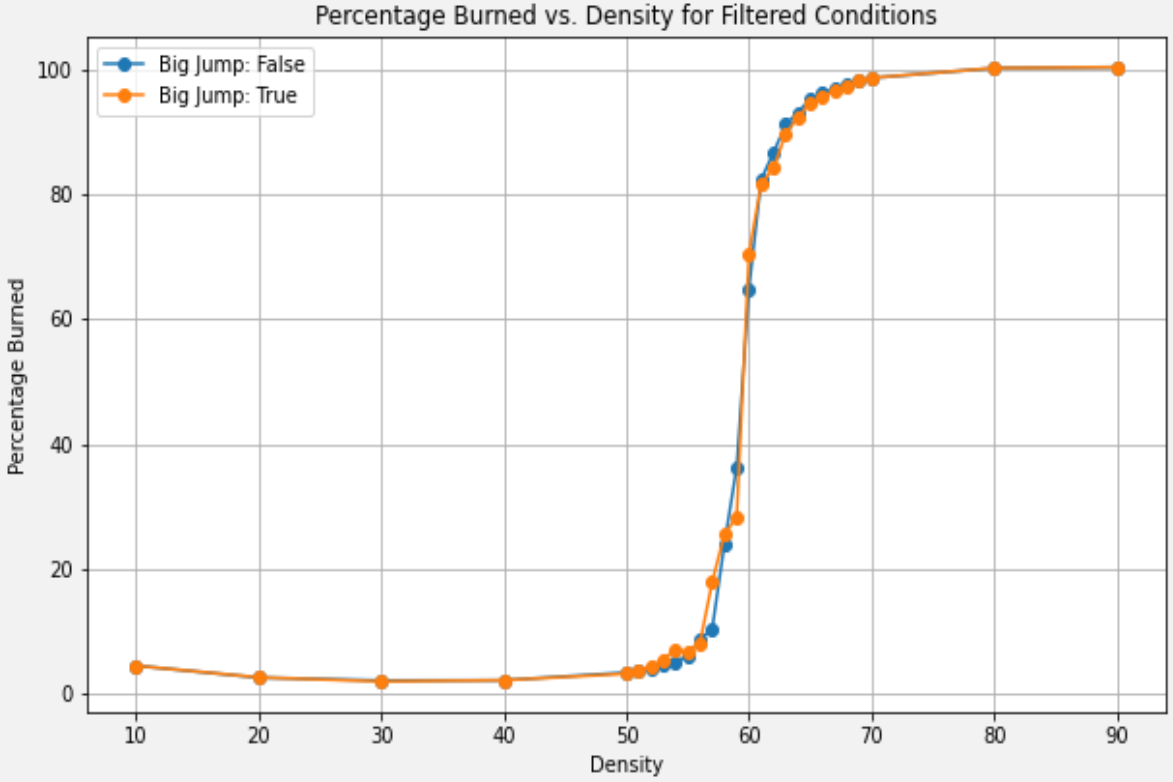
GRAPH OF BURNT PERCENTAGE ACCORDING TO DENSITY WITH GRID OF SIZE 901*901 (BY CHANGING WEST-WIND SPEED)



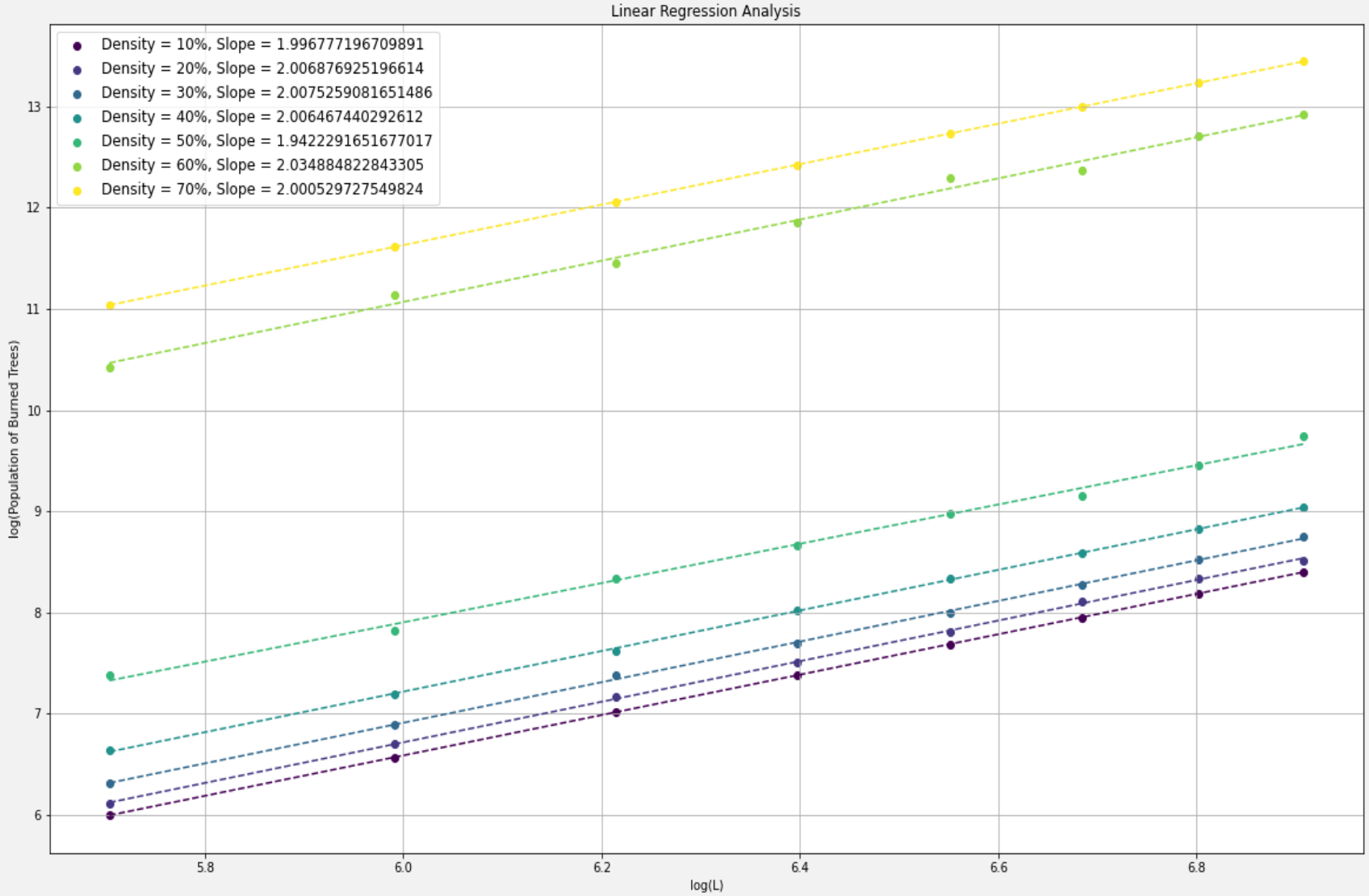


GRAPH OF BURNT PERCENTAGE ACCORDING TO DENSITY WITH GRID OF SIZE 901*901 (BY CHANGING BIG-JUMP)

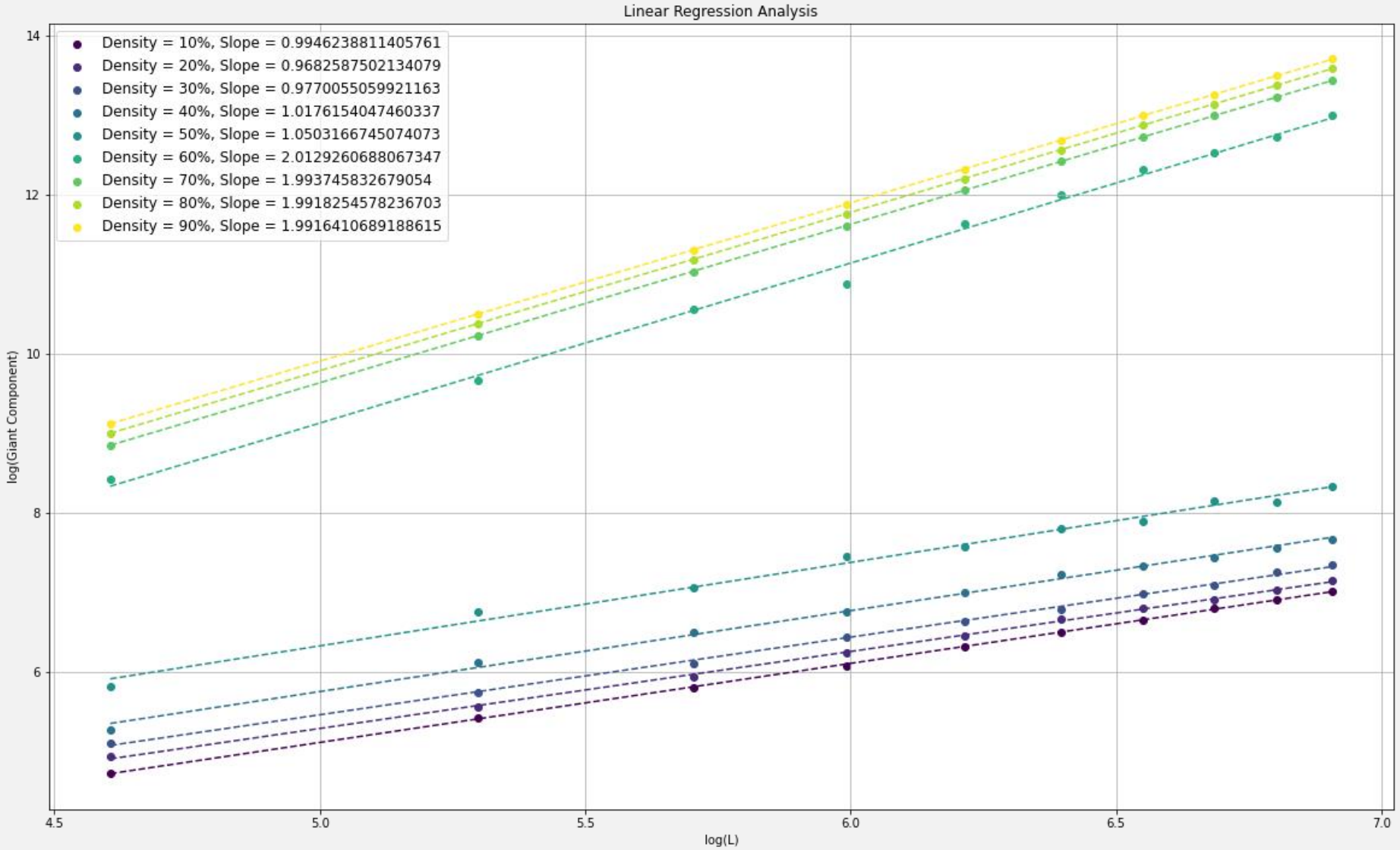




AN APPROXIMATION OF THE FRACTAL DIMENSION



FRACTAL DIMENSION



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