Few-body Systems in Nuclear Physics

a. Anti-kaonic Nuclear Systems
b. Few-body Faddeev Method

Sajjad Marri
Isfahan University of Technology

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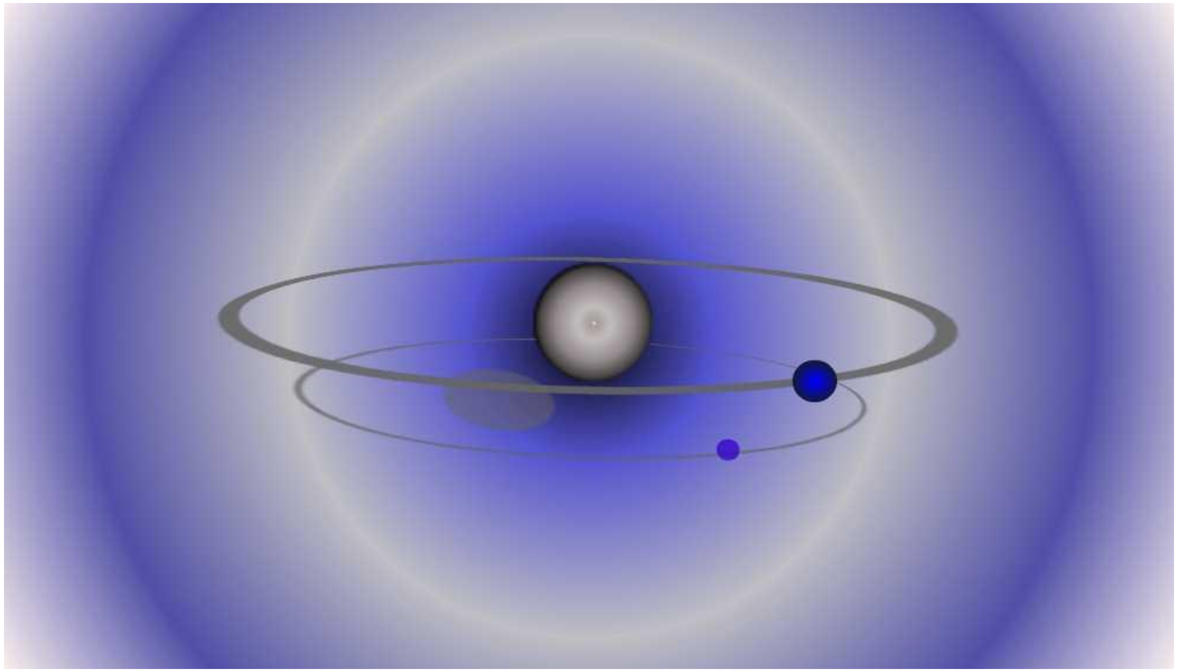
Outline

- Exotic atom and nuclei
- The $\bar{K}N$ interaction
- Antikaonic nuclear bound states
- Faddeev method
- Kaonic systems in production reactions

Exotic atoms (Kaonic Hydrogen)
A key experiment in $K_{bar}N$ interaction

- Exotic atoms are QED bound systems
- A negatively charged particle, other than an electron, orbits a nucleus
- The principal interaction with the nucleus is electromagnetic.



Exotic atoms as probes for strong interaction at threshold

Exotic atoms (Kaonic Hydrogen) A key experiment in $K_{bar}N$ interaction

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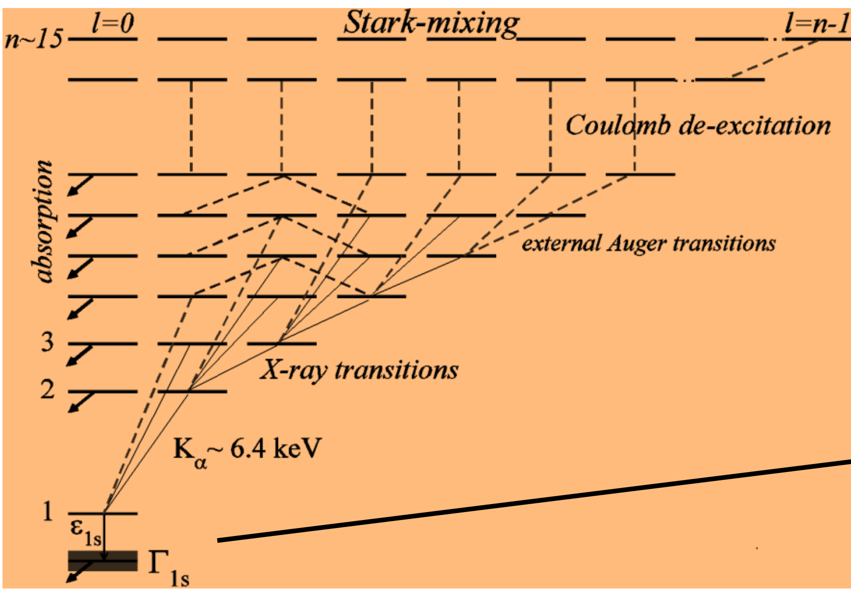
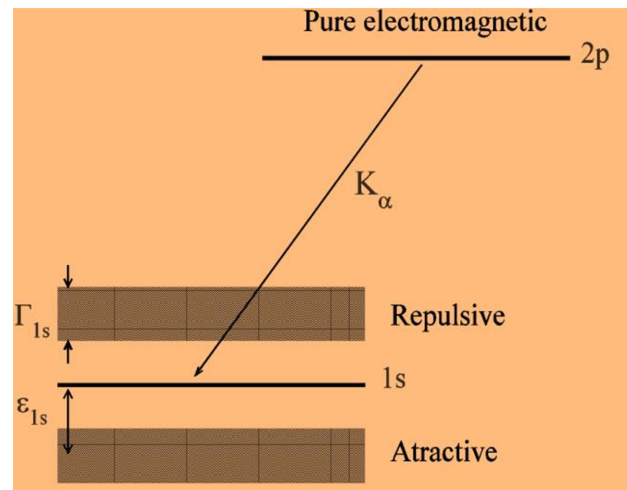
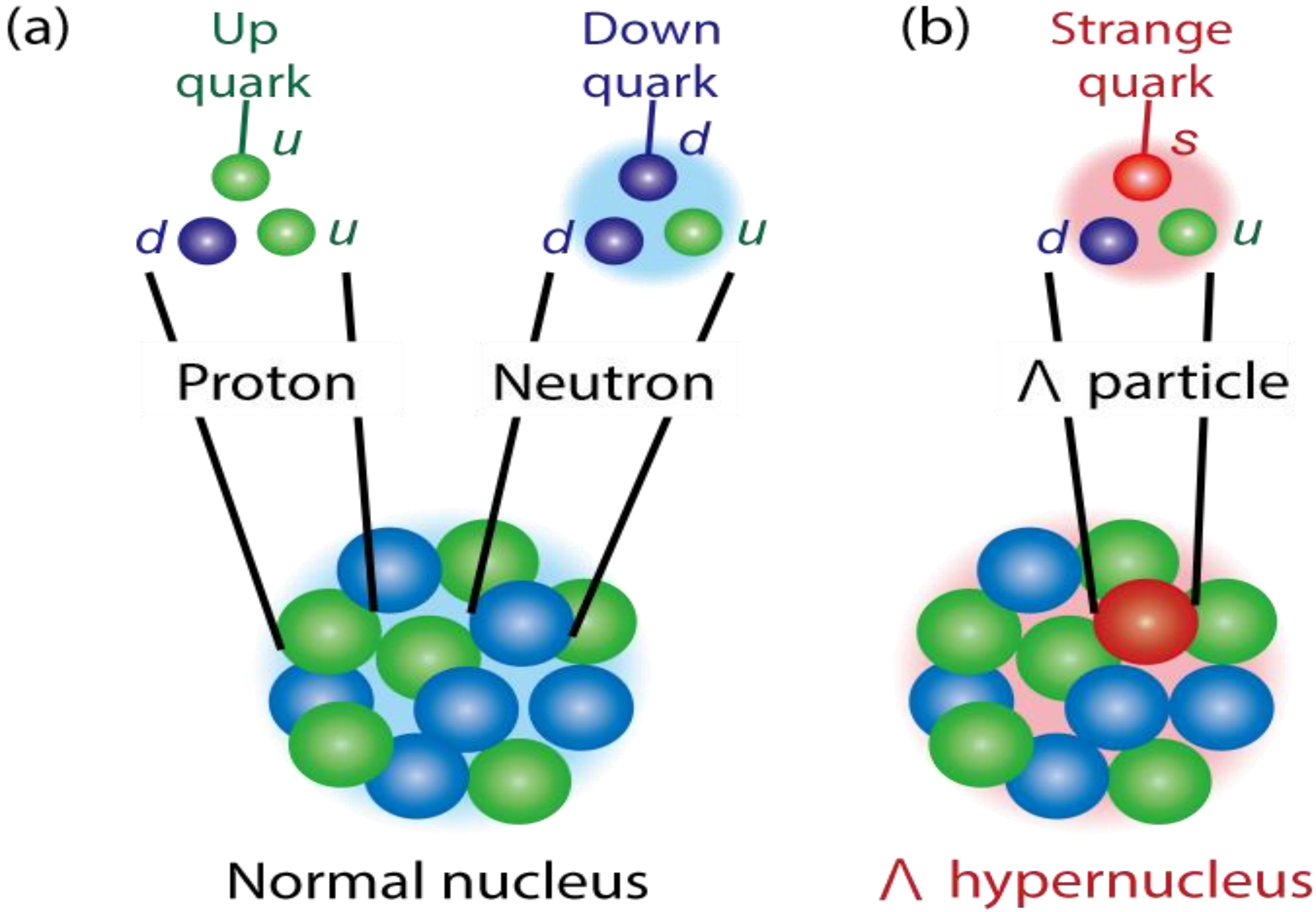




K nuclei… Exotic system !



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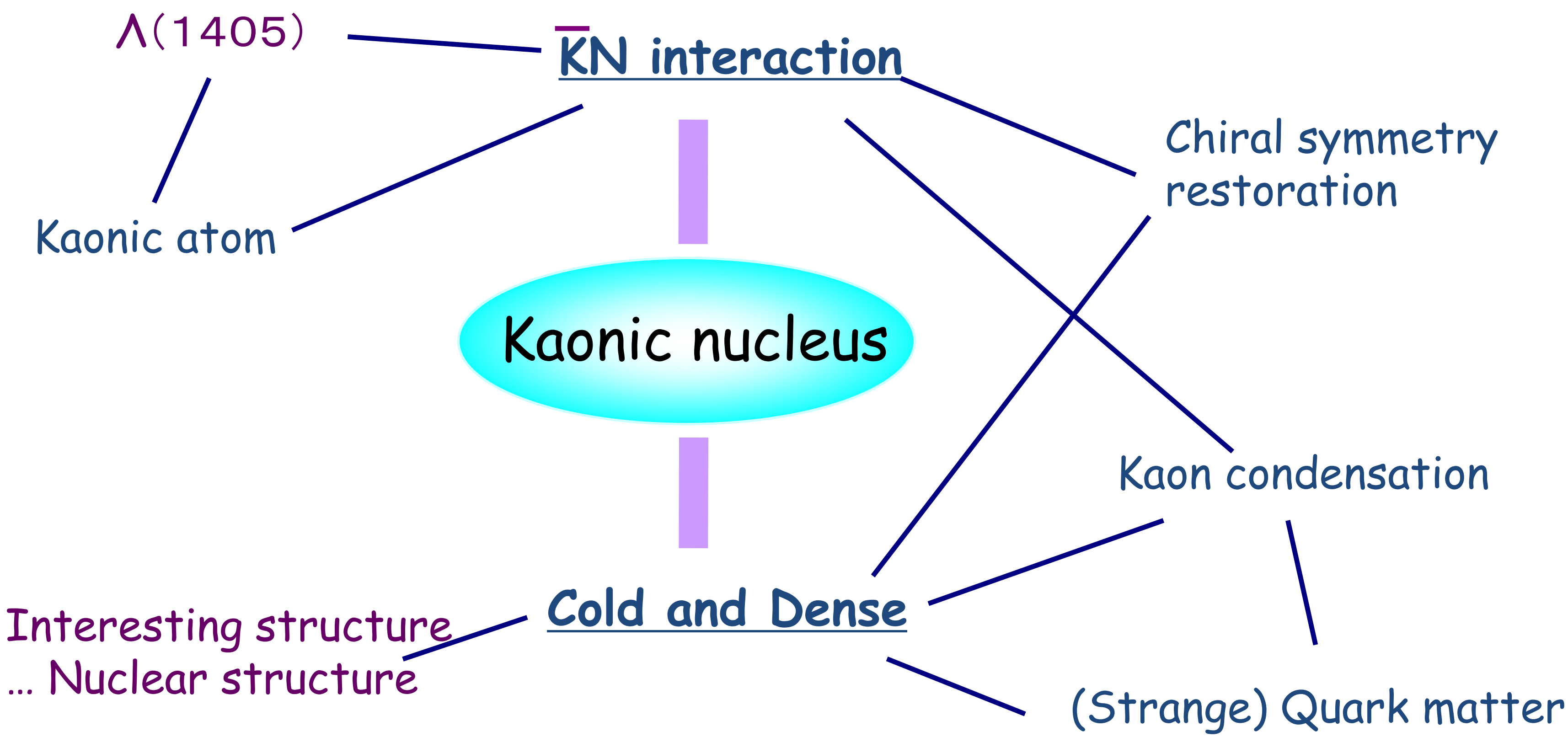
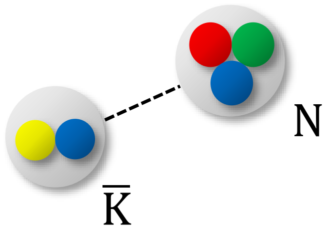


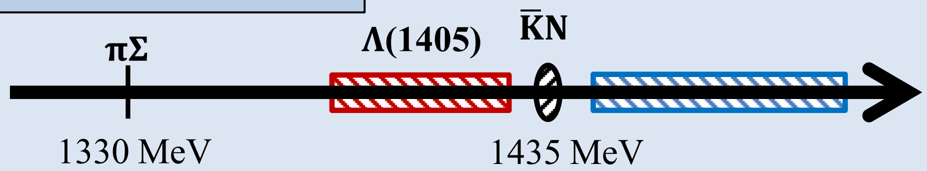
… related to various fields!!

Experimental constraints on kaon-nucleon interaction

In order to understand the structure of Λ(1405), precise determination of $\bar{K}N - \pi\Sigma$ (I=0) interaction is necessary.



experimental constraint



above KN threshold energy: - K^-p cross section

at/just-below KN threshold energy: - Branching ratio

- kaonic atoms(new data by SIDDHARTA)

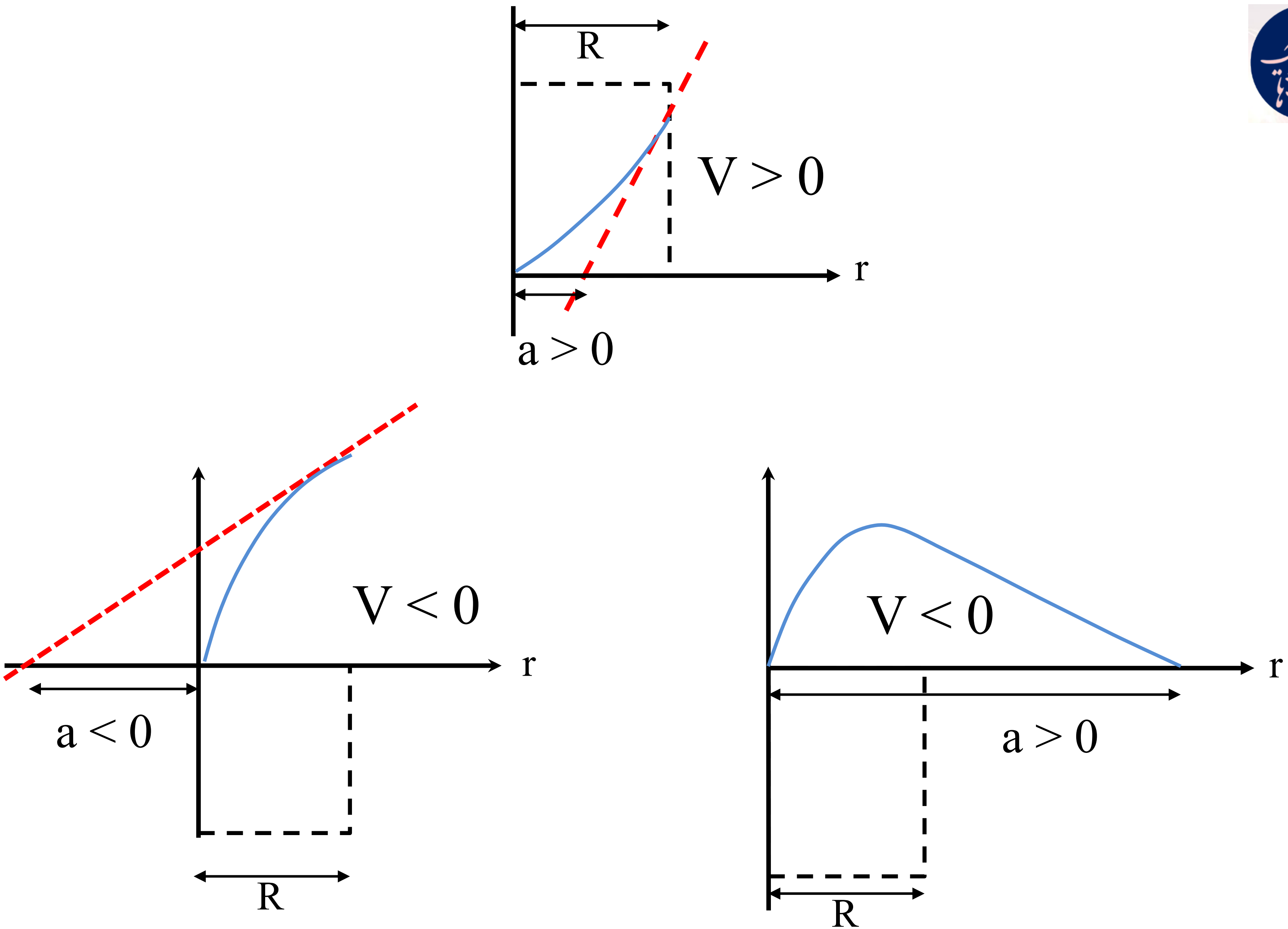
below the KN threshold energy:

-So far, cannot perform πΣ elastic scattering experimentally

- πΣ mass spectra from Λ(1405) production reactions

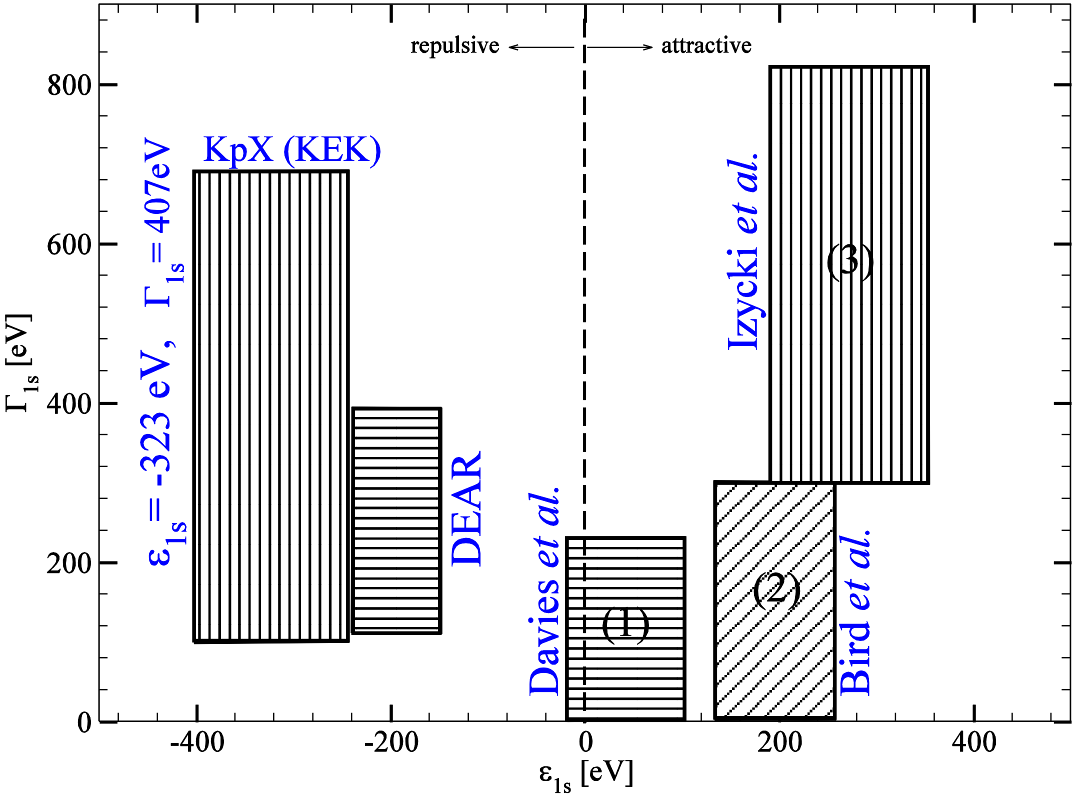
(CLAS, LEPS, HADES, J-PARC)

potentials... Scattering length!



Kaonic Hydrogen pozzle

- Most precise values for shift and width from DEAR experiment
- But still precision limited (e.g. error bar of width > 50%)
- shift vs. width (1 σ errors) see below



DEAR (2005)

ε_{1s} = - 193 ± 37 (stat.) ± 6 (syst.) eV

Γ_{1s} = 249 ± 111 (stat.) ± 30 (syst.) eV

$a_{K\text{-}p}$ = (-0.468 ± 0.090 ± 0.015) + i(0.302 ±0.135± 0.036) fm

KpX (1998)

ε_{1s} = - 323 ± 63 (stat.) ± 11 (syst.) eV

Γ_{1s} = 407 ± 208 (stat.) ± 100 (syst.) eV

$a_{K\text{-}p}$ = (-0.78 ± 0.15 ± 0.03) + i(0.49 ±0.25± 0.12) fm

using Deser-Trueman (i.e. lowest order)

Reliable theory has to be consistent with these informations

Starting from:

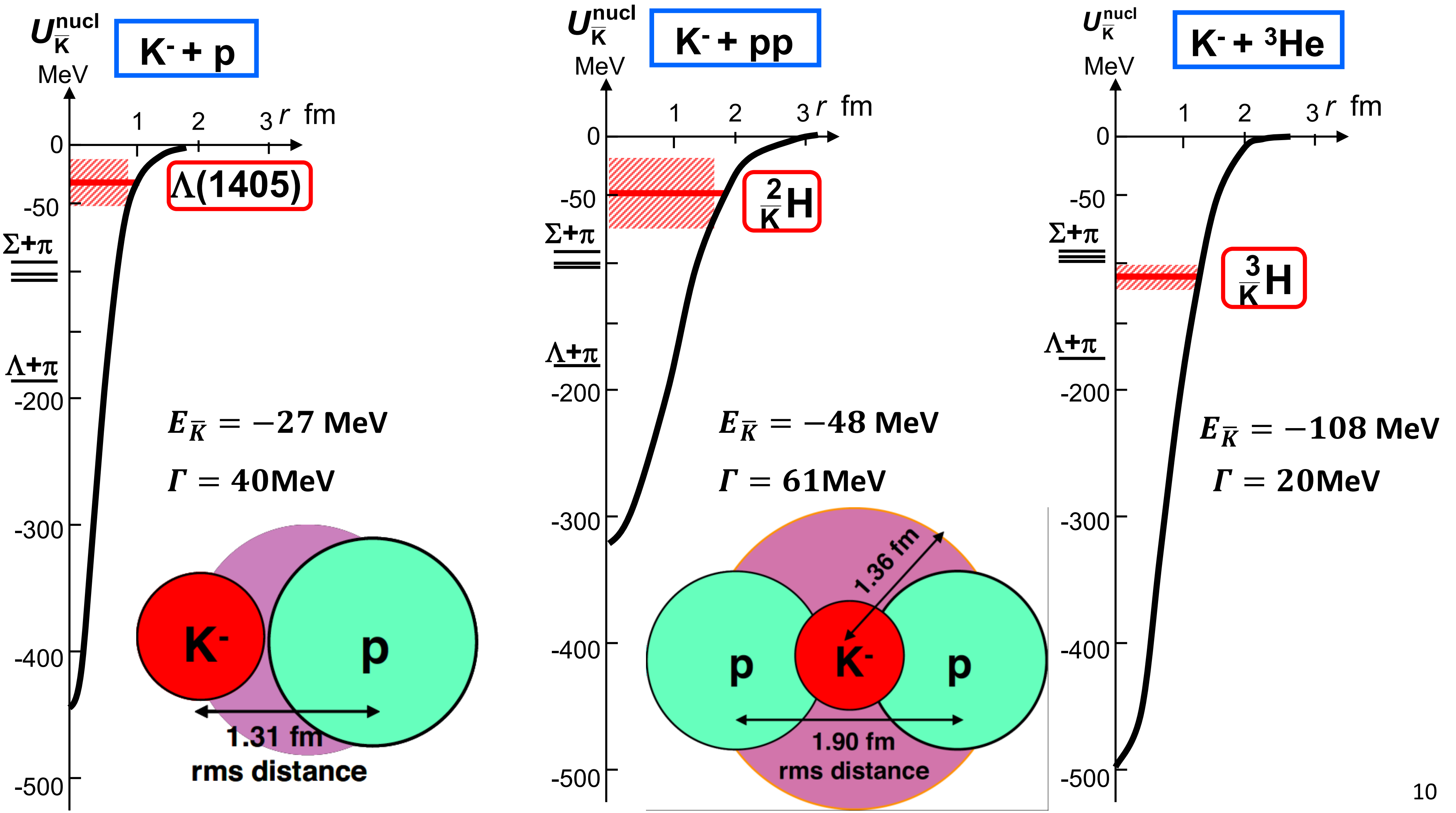
- $K^- p$ atom
- $\bar{K}N$ scattering
- $\Lambda(1405)$

Strong $K^- - p$ attraction (Weise:1996)

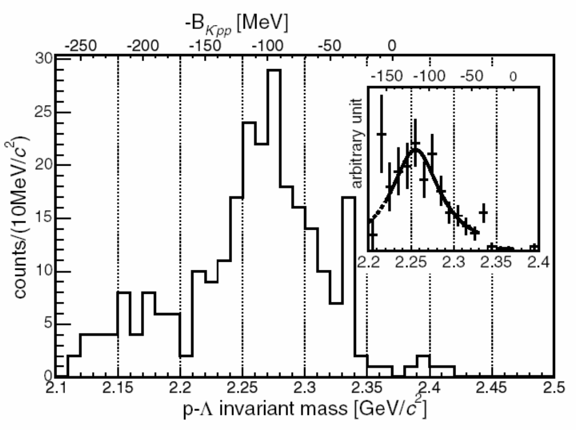
Nuclear shrinkage predicted 2002

Y. Akaishi and T. Yamazaki, PRC 65 (2002) 044005

T. Yamazaki and Y. Akaishi, PLB 535 (2002) 70



Evidence for $(K^{-}pp)$ by FINUDA @ DaΦne



M. Agnello *et al.*, PRL 94, 212303 (2005)

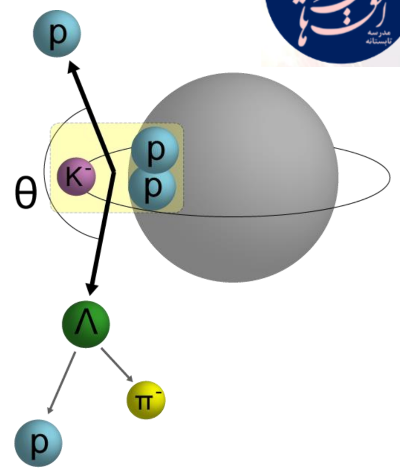
$$e^{-} + e^{+} \rightarrow \Phi \rightarrow K^{-} + K^{+}$$

$$K^{-} + A \rightarrow (K^{-}pp) + X \rightarrow \Lambda + p + X$$

V.K. Magas, E. Oset, *et al.*, nucl-th/0601013

M=2.255±0.009 GeV

$\Gamma=67^{+14+2}_{-11-3}$ **MeV**



reanalysis of old DISTO data

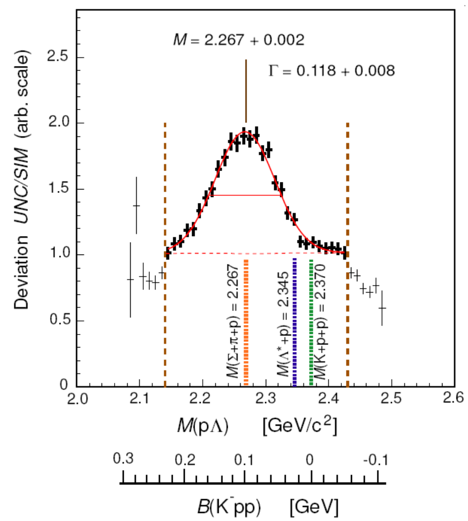
T. Yamazaki *et al.*, Phys. Rev. Lett. **104**, 132502 (2010).

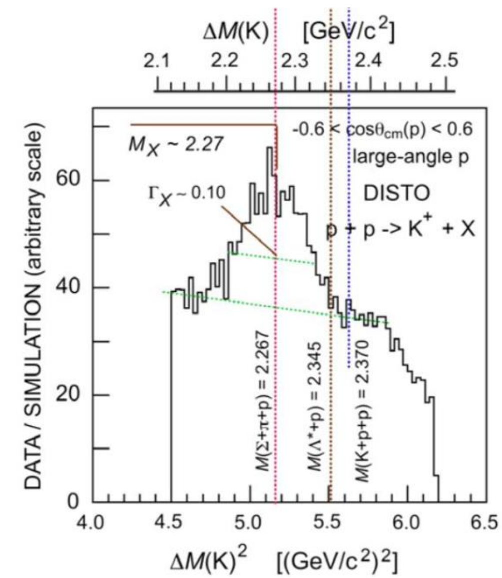
$$p + p \rightarrow K^{+} + X$$

$$\rightarrow K^{+} + \Lambda + p \text{ at } 2.85\text{GeV}$$

M=2.265±0.002 GeV

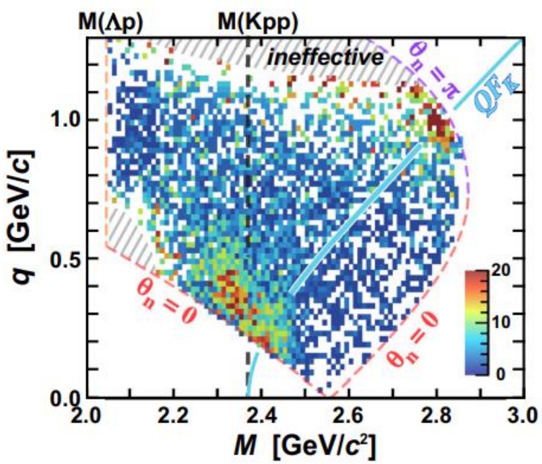
Γ**=118±0.008 MeV**

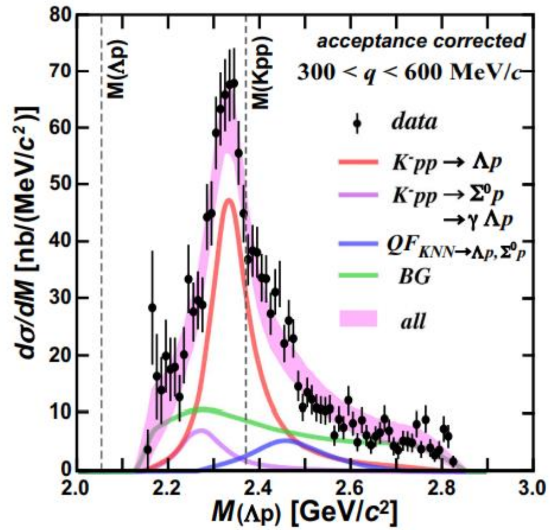


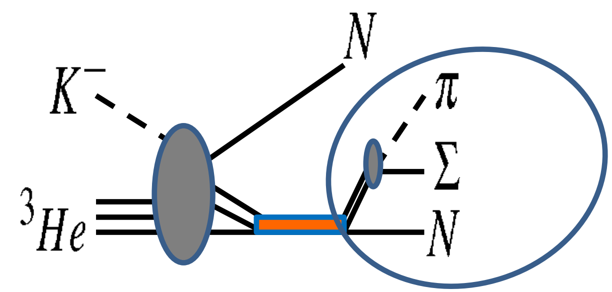


proving K⁻pp → strongly bound, dense

signal of $K^{-}pp$ in production reactions … JPARC-E15







PHYSICAL REVIEW C **102**, 044002 (2020)

Observation of a $\bar{K}NN$ bound state in the ^{3}He(K^{-}, Λp)n reaction

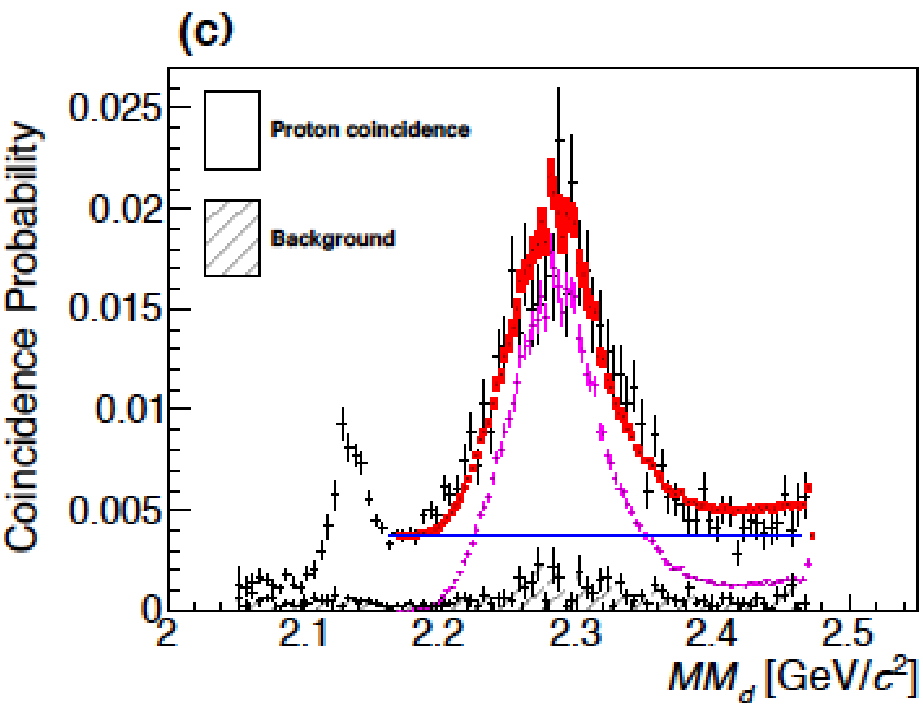
T. Yamaga,[1,*] S. Ajimura,[2] H. Asano,[1] G. Beer,[3] H. Bhang,[4] M. Bragadireanu,[5] P. Buehler,[6] L. Busso,[7,8] M. Cargnelli,[6] S. Choi,[4] C. Curceanu,[9] S. Enomoto,[14] H. Fujioka,[15] Y. Fujiwara,[12] T. Fukuda,[13] C. Guaraldo,[9] T. Hashimoto,[20]

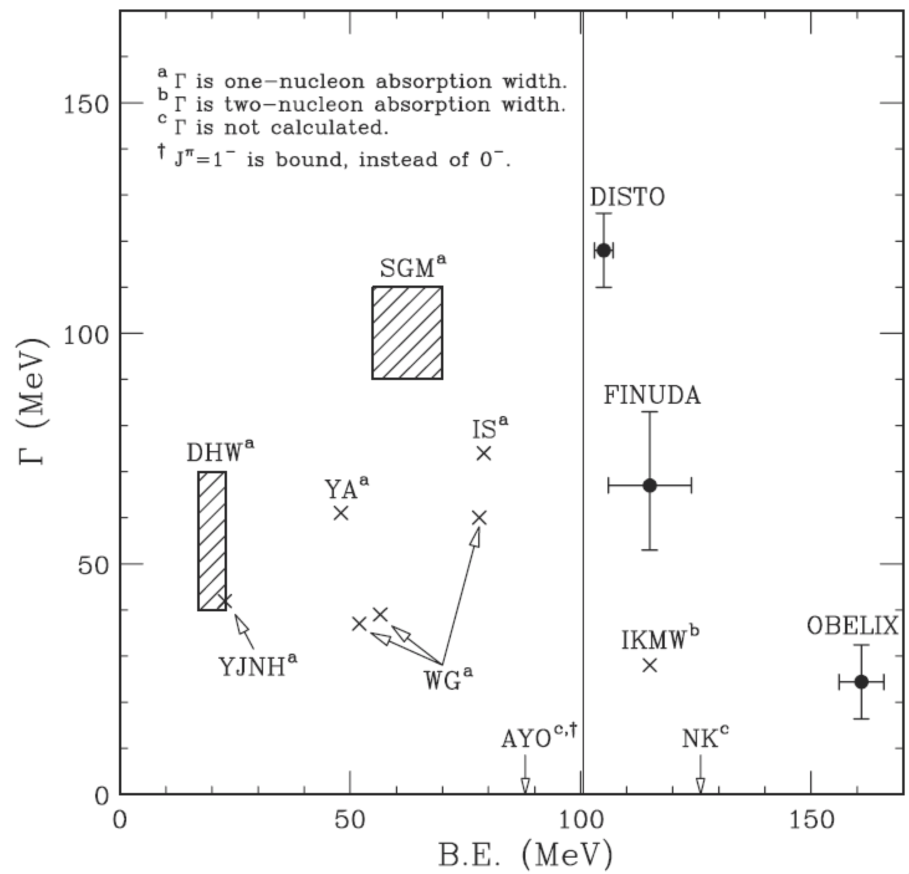
J-PARC E27 results

PTEP

Prog. Theor. Exp. Phys. **2013**, 00000 (8 pages)
DOI: 10.1093/ptep/0000000000

Observation of the "K^-pp"-like structure in the $d(\pi^+, K^+)$ reaction at 1.69 GeV/c





B(K-pp) = 95 $^{+18}_{-17}$ $^{+30}_{-21}$ MeV

To solve a two-body system

- **Schrodinger Equation with boundary conditions**

$$H\psi = E\psi$$

- **Lippmann-Schwinger Equation with boundary conditions**

$$\mathrm{T} = V + VGT$$

To solve an n-body system

Faddeev method:

A technique to convert the Schrodinger equation with boundary conditions for a three-body system to a set of integral equation

- **The theoretical foundation of the three-body problem was carried out by Faddeev**

  L. D. Faddeev, sov. Phys, JETP 12 1014 (1961)

- **After important reformulation by Alt, Grassberger and sandhas and Glokle**

  E. O. Alt et al, Nucl. Phys. B2, 167 (1967)
  W. Glokle, Nucl. Phys. A141, 620 (1970)

the kernel of Faddeev eq. is compact, Which guarantees the convergence and uniqueness of solution.

Faddeev Yakubovsky Method	Faddeev AGS Method
O.A. Yakubovsky, Yad. Fiz. 5, 1312 (1967)	P. Grassberger and W. Sandhas, Nucl. Phys. B2, 181 (1967). E.O. Alt, P. Grassberger, and W. Sandhas, PRC 1, 85 (1970).
Wave function and binding energy	**Scattering amplitudes and binding energy**

Why Faddeev method?

The usual LS equation does not have a unique solution for scattering state in three-body problem.

$$\langle \vec{k}'_a \vec{p}'_a | G_0(z) V_a | \vec{k}_a \vec{p}_a \rangle = \delta(\vec{p}'_a - \vec{p}_a) \left\langle \vec{k}'_a \middle| G_0\left(z - \frac{p^2}{2\upsilon_a}\right) V_a \middle| \vec{k}_a \right\rangle$$

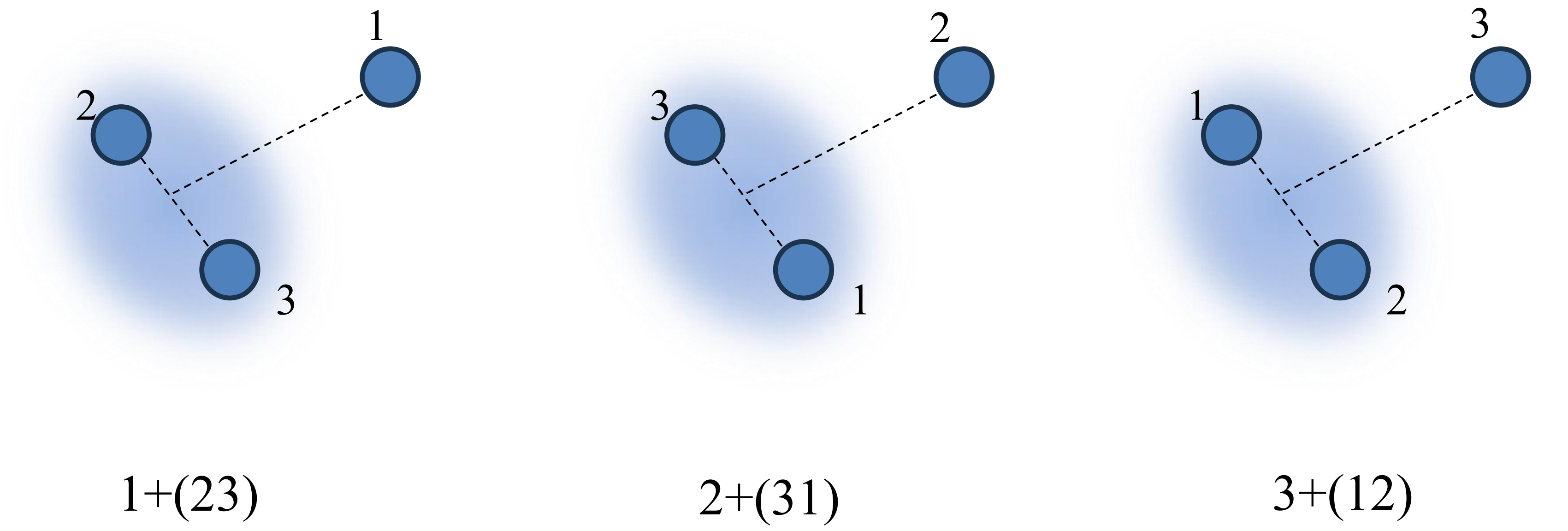
This fact prevents the kernel $K = G_0(z)V_a$ from being a Hilbert-Schmidt operator. That is, its Schmidt-norm (trace of $KK^{\dagger}$)

$$|K|^2 = \int d\vec{k}' d\vec{p}' d\vec{k} d\vec{p} \left| \langle \vec{k}'_a \vec{p}'_a | G_0(z) V_a | \vec{k}_a \vec{p}_a \rangle \right|^2$$

is not a finite quantity (the kernel is not square integrable). Thus, standard methods of solution of Fredholm theory can not be applied as in the two-body case.

Faddeev partitions

Three-body case:

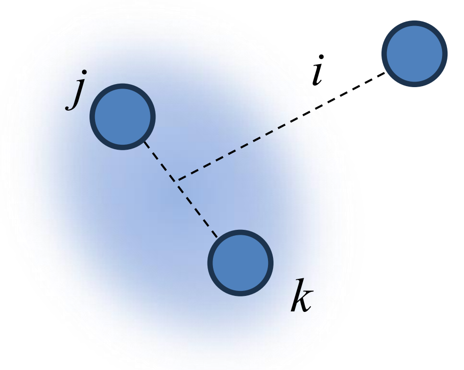


Faddeev AGS equations

- $U_{ij} = (1-\delta_{ij})G_\circ^{-1} + \sum_{k=1}^{3}(1-\delta_{ik})\, T_k G_\circ U_{kj}$

U_{ij} is the Faddeev transition operators

$j+(ki)\rightarrow i+(jk)$



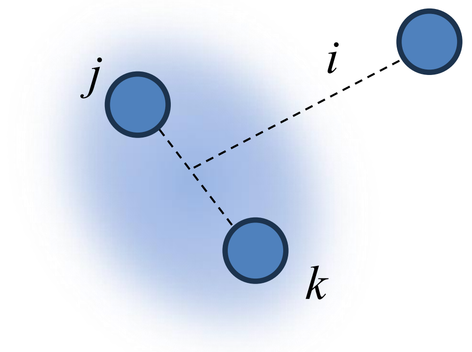
$i+(jk)$

- $T_i = V_i + G_0 V_i T_i$

Faddeev Yakubovsky equations

$$\left(H_0 + \sum_{i=1}^{3} V_i\right)|\Psi\rangle = E|\Psi\rangle$$

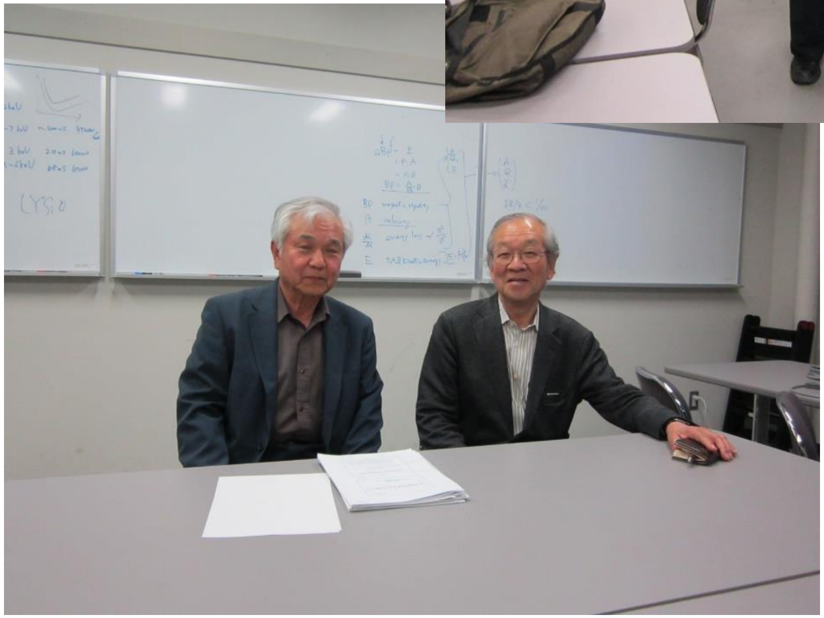
$$|\Psi\rangle = \sum_{i=1}^{3} |\Psi\rangle_i$$



$$|\Psi\rangle_i = G_0 t_i \sum_{j\neq i}^{3} |\Psi\rangle_j$$

$$t_i = V_i + G_0 V_i t_i$$

Our contribution to this field …



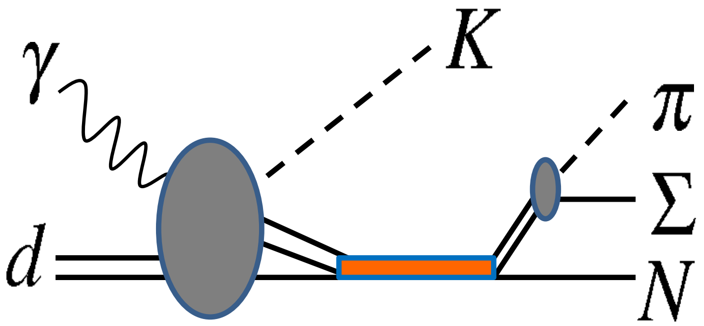
Our contribution to this field …

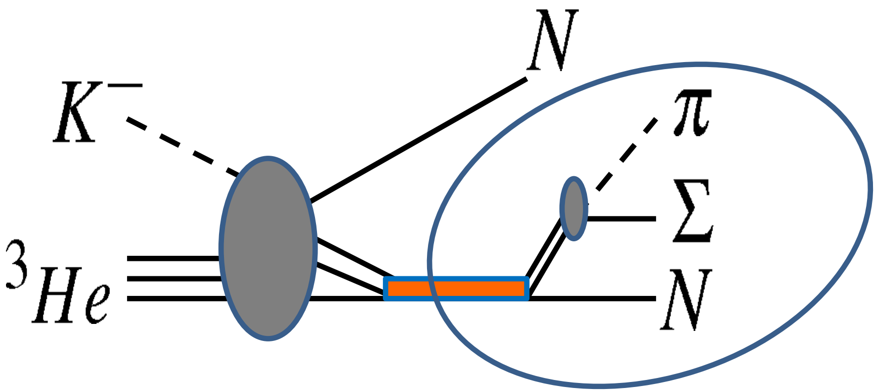
- The suitable scenario to describe the Λ(1405)!
- The structure of the K^{bar} Nuclear Clusters!!

1. S. Marri, S. Z. Kalantari and J. Esmaili, "*Investigation of kaon-deuteron interaction and the structure of Λ(405) resonance using Faddeev method*" Iranian Journal of Physics Research, 18 (4), 539-549 (2019).
2. J. Esmaili, M. Daneshmand and S. Marri, "*The study of Λp invariant mass spectra comes from In-flight kaon interaction on deuteron*" Iranian Journal of Physics Research, 19 (4), 691-697 (2020).
3. Jafar Esmaili, Sajjad Marri, Morteza Raeisi and Ahmad Naderi Beni, "*Trace of Λ(1405) resonance in low energy $K^- + {}^3He \rightarrow (\pi\Sigma)^0 + d$ reaction*", Eur. Phys. J. A (2021) 57: 120.
4. S. Marri, M. N. Nasrabadi and S. Z. Kalantari, "*Structure of Λ(1405) resonance and $\gamma p \rightarrow K^+ + (\pi\Sigma)^0$ reaction*", Physical Review C, 103, 055204 (2021).

1. S. Marri and S. Z. Kalantari, "*Coupled-channels Faddeev AGS calculation of K^-ppn and K^-ppp quasi-bound states*", Eur. Phys. J. A, 52, 282 (2016).
2. S. Marri, S. Z. Kalantari and J. Esmaili, "*Deeply quasi-bound state in single- and double-$\bar{K}$ nuclear clusters*", Eur. Phys. J. A, 52, 361 (2016).
3. S. Marri, "*Structure, formation and decay of $\bar{K}NN$ by Faddeev-AGS calculations*", Chinese Physics C Vol. 43, No. 6 (2019) 064101.
4. S. Marri and J. Esmaili "*Three and four-body kaonic nuclear states: Binding energies and widths*", Eur. Phys. J. A (2019) 55: 43.
5. S. Marri, S. Z. Kalantari and J. Esmaili, "*Investigation of $\bar{K}\bar{K}N$ coupled channel system by Faddeev method*" Iranian Journal of Physics Research, 18 (2), 291-299 (2018).
6. S. Marri, "*Signature of N^* resonance in mass spectra of the $\bar{K}KN$ decay channels*", Physical Review C, 102, 015202 (2020).
7. S. Marri and M. N. Nasrabadi, "*Solution of the Faddeev equations for $K\bar{K}NN$ quasi-bound state using the quasi-particle method*", Eur. Phys. J. A (2022) 58: 130.
8. S. Marri, "*Signature of K^-pp quasi-bound state in production reaction*" prepared for submission to PRC.

signal of $K^{-}pp$ in production reactions





Purpose of the work: within Faddeev approach

- signal of strange dibaryon resonances
- dynamics of $\bar{K}N - \pi\Sigma$ in resonance production reaction

با تشکر از توجه شما

Kaonic Nuclear Clusters contribution to fundamental physics

- information concerning the *modification of the kaon mass and of the $\bar{K}N$ interaction in the nuclear medium* => interesting and important from the viewpoint of the spontaneous and explicit symmetry breaking of *QCD*

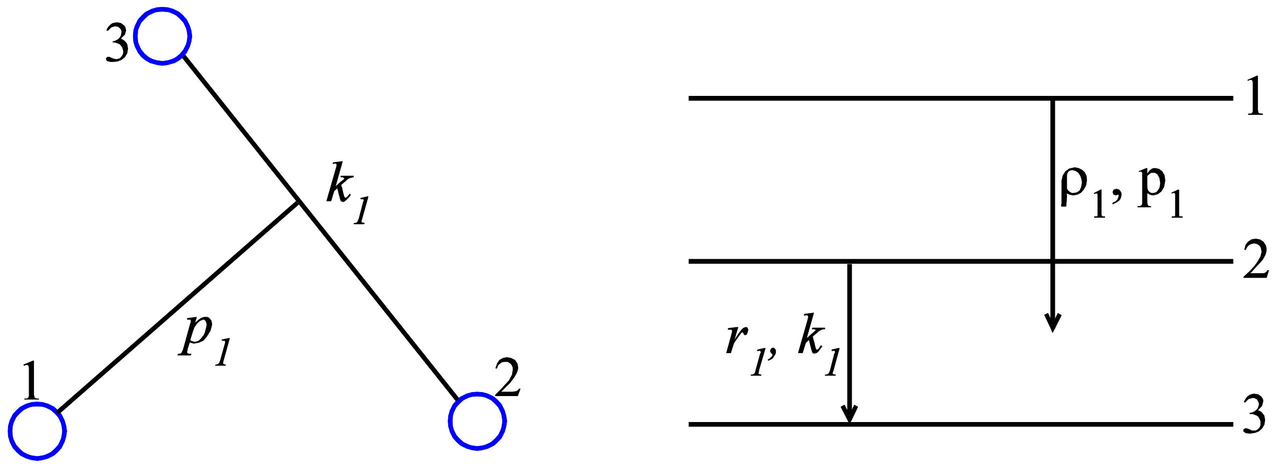
- information on the *transition from the hadronic phase to a quark-gluon phase* => changes of *vacuum properties of QCD* and *quark condensate*

- kaon condensation in nuclear matter => implications in *astrophysics (neutron stars, strange stars)*

- *nuclear dynamics under extreme conditions (nuclear compressibility, etc) could be investigated*

Faddeev partitions:

Variables. Kinematic transformations



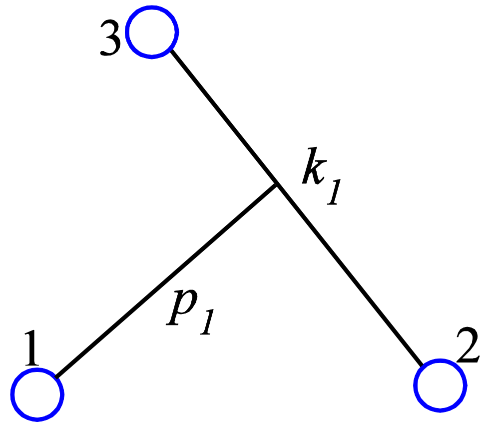
$$\vec{r}_1 = \vec{R}_2 - \vec{R}_3 \,, \qquad \vec{k}_1 = \mu_1 \frac{d\vec{r}_1}{dt} = \frac{m_3\vec{K}_2 - m_2\vec{K}_3}{m_2+m_3}$$

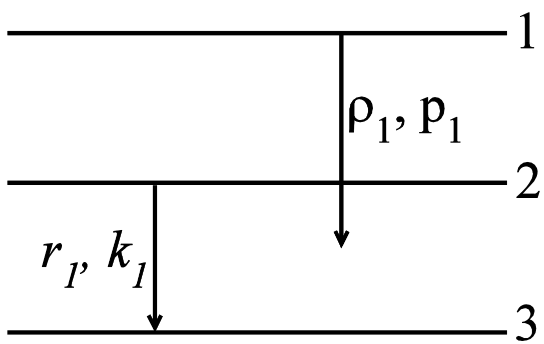
$$\vec{\rho}_1 = \vec{R}_1 - \frac{m_2\vec{R}_2 + m_3\vec{R}_3}{m_2+m_3} \,, \qquad \vec{p}_1 = \nu_1 \frac{d\vec{\rho}_1}{dt} = \frac{(m_2+m_3)\vec{K}_1 - m_1\left(\vec{K}_2+\vec{K}_3\right)}{m_2+m_3}$$

$$\mu_1 = \frac{m_2 m_3}{m_2+m_3} \,, \qquad \nu_1 = \frac{m_1(m_2+m_3)}{m_1+m_2+m_3}$$

Faddeev partitions:

Variables. Kinematic transformations





$$h_0 = \frac{k^2{}_a}{2\mu_a} + \frac{p^2{}_a}{2\nu_a}$$

$$\langle \vec{k}'_a \vec{p}'_a | \vec{k}_a \vec{p}_a \rangle = \delta(\vec{k}'_a - \vec{k}_a)\delta(\vec{p}'_a - \vec{p}_a)$$

$$\langle \vec{k}'_b \vec{p}'_b | \vec{k}_a \vec{p}_a \rangle = \delta\left(\vec{k}_a + \frac{m_b}{m_b + m_c}\vec{k}'_b - \frac{m_c(m_a + m_b + m_c)}{(m_a + m_c)(m_b + m_c)}\vec{p}'_b\right) \times \delta\left(\vec{p}_a + \vec{k}'_b + \frac{m_a}{m_a + m_c}\vec{p}'_b\right)$$