Bose-Einstein condensation of a rotating Bose gas in the presence of a background magnetic field

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Jan. 27-28, 2026
16th Conference on Physics of Fields and Particles - IUT

Pion Condensation in RHICs

- **Meson Production**
  - Hot, dense QGP from collisions hadronizes
  - Pions (bosons) are the most abundant product
- **BEC Conditions**
  - Requires very high pion density
  - Achieves near-zero relative momentum in expansion
- **Link to QGP & Data**
  - Forms during hadronic phase after QGP cools
  - **Experimental signatures:** enhanced low momentum yield, suppressed correlations (ALICE). **[arXiv:2212.09288]**

Effects of Rotation (Vorticity) and Magnetic Field

- ★ **Rotation** $[10^{20} - 10^{22}\mathrm{Hz}]$
  - ★ Λ spin polarization
  - ★ Chiral vortical effect
  - ★ Reduction of scalar condensate
  - ★ ...
- ★ **Magnetic Field** $[10^{18}\mathrm{G}]$
  - ★ Chiral magnetic effect
  - ★ Chiral seperation effect
  - ★ (Inverse) Magnetic catalysis of χSB
  - ★ ...

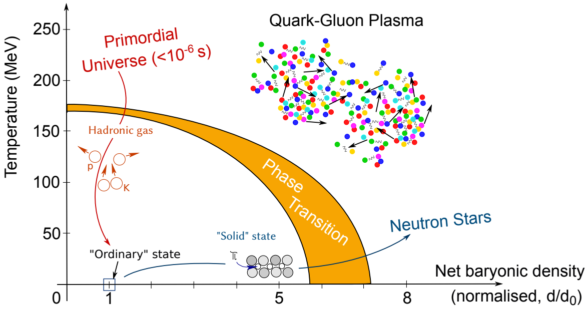
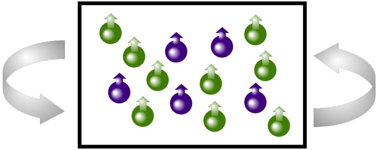


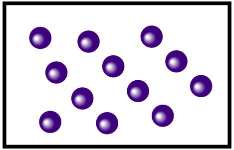
Figure: QCD phase diagram. © 2011 CERN, for the benefit of the ALICE Collaboration

Rotation Vs. Chemical Potential



Rotation

$H = H_0 - \Omega J_z$



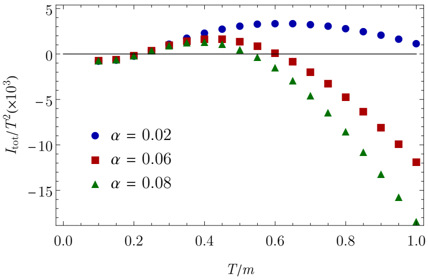
Chemical Potential

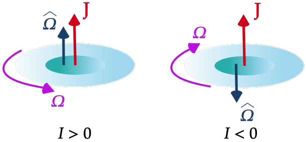
$H = H_0 - \mu N$

★ For a massless Dirac fermion at the axis of rotating unbounded system **[XG Huang Talk]**

$$P = \frac{7\pi^2 T^4}{180} + \frac{(\Omega/2)^2 \, T^2}{6} + \frac{(\Omega/2)^4}{12\pi^2}, \qquad P = \frac{7\pi^2 T^4}{180} + \frac{\mu^2 T^2}{6} + \frac{\mu^4}{12\pi^2} \tag{1}$$

The Effect of Rigid Rotation on (Spin-0) Bose Gas





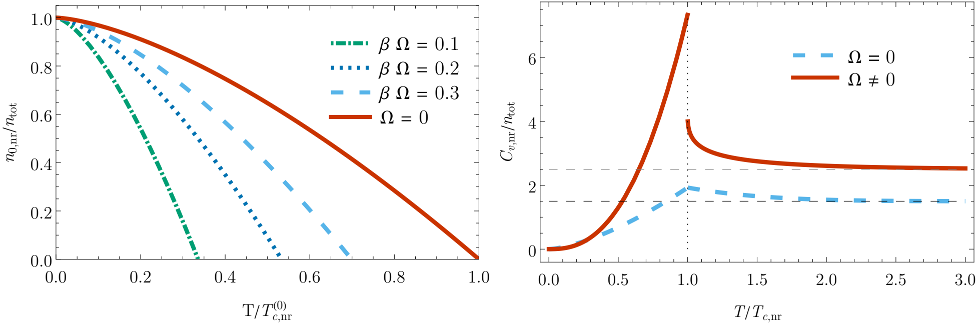
★ Supervortical $T(\alpha) \Rightarrow I = 0$

★ Negative moment of inertia **[arXiv:2405.09481]**

$$\mathbf{J} \parallel \hat{\mathbf{\Omega}} \Rightarrow I > 0$$

$$\mathbf{J} \nparallel \hat{\mathbf{\Omega}} \Rightarrow I < 0$$

BEC in Rotating Frame



- ★ Rotation decreases T_c.
- ★ Rotation changes order of phase transition. **[arXiv:2411.12581,arXiv:2508.17055]**

Complex Klein-Gordon Field in Presence of EM + Rot

★ In rotating frame, the Lagrangian density is given by

$$\mathcal{L} = \left|(\partial_t - i\Omega L_z)\varphi\right|^2 - \left|D_i\varphi\right|^2 - m^2\left|\varphi\right|^2 \tag{2}$$

In a rotating frame defined by $A^\mu = \frac{B_0}{2}(-\Omega r^2, -y, x, 0)$, the covariant derivative is $D_\mu = \partial_\mu + ieA_\mu$ and $L_z = -i\partial_\theta$.

★ Equation of motion

$$\varphi_{n\ell}(x) = e^{-iEt + i\ell\theta + ik_z z} f_{n\ell}(r) \tag{3}$$

where radial solution obtained in terms of Laguerre polynomial

$$f_{n\ell}(r) = \sqrt{\frac{(eB_0)^{|\ell|+1}}{2^{|\ell|}}\frac{n!}{(n+\ell)!}}\, e^{-eB_0 r^2/4} r^{|\ell|} L_n^{|\ell|}\left(\frac{eB_0 r^2}{2}\right) \tag{4}$$

Energy Spectrum and Fock-Schwinger Two Point Function

★ Energy spectrum in rotating frame

$$\tilde{E} = \sqrt{k_z^2 + m^2 + eB_0(2n+1)} - \ell\Omega \tag{5}$$

where $n = 0, 1, 2, \cdots$ indicate Landau's levels.

★ In generalized Fock-Schwinger approach, two point function is expressed as

$$\mathcal{G}(x, x') = -i \int_{-\infty}^{.0} \mathrm{d}\tau \sum_{\Lambda} e^{-i\Lambda\tau} \varphi_\Lambda(x) \varphi_\Lambda^*(x') \tag{6}$$

where τ denotes the proper-time parameter and $\Lambda = -\tilde{E}^2 + k_z^2 + (2n+1)eB_0 + m^2$.

★ Propagator in momentum space

$$\mathcal{G}_{n\ell,n'\ell'}(p, p') = (2\pi)^2 \widehat{\Delta}_{n\ell,n'\ell'}(p, p') \mathcal{G}_{n\ell}(p) \tag{7}$$

where $\widehat{\Delta}_{n\ell,n'\ell'}(p, p') \equiv \delta(p_0 - p_0')\delta(p_z - p_z')\delta_{\ell\ell'}\delta_{nn'}$.

Propagator

★ Propagator at $T = 0$

$$\mathcal{G}_{n\ell}(p) = \frac{1}{-(p_0 + \ell\Omega)^2 + \omega_B^2}, \quad \omega_B^2 = p_z^2 + eB_0\,(2n+1) + m^2 \tag{8}$$

★ Propagator at $T \neq 0$ (Imaginary time formalism, $p_0 \to i\omega_n + \mu$)

$$\mathcal{G}_{n\ell}(p) = \frac{1}{(\omega_n + i\mu_{\text{eff}})^2 + \omega_B^2}, \quad \mu_{\text{eff}} \equiv \mu + \ell\Omega \tag{9}$$

where $\omega_n = 2\pi nT$ is bosonic Matsubara frequency.

Thermodynamic Potential

★ Thermodynamic potential in grand-canonical ensemble

$$\Phi = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \ln\left[\beta^2 \mathcal{G}_{n\ell}^{-1}(p)\right] \tag{10}$$

★ In presence of $B + \Omega$ and imaginary time formalism

$$\begin{aligned} \int \frac{\mathrm{d}p_0}{2\pi} \quad &\to \quad T\sum_{\omega_n}, \\ \int \frac{\mathrm{d}p_x\,\mathrm{d}p_y}{(2\pi)^2} \quad &\to \quad \frac{eB_0}{2\pi}\sum_{n=0}^{\infty}\sum_{\ell=-n}^{N-n} \end{aligned} \tag{11}$$

where $N = \lfloor eB_0 R^2/2 \rfloor$ is Landau degeneracy factor.

Thermal Part of Thermodynamic Potential in LLL Approximation

★ LLL + $\mu_\Omega(= N\Omega) \to$ thermal part of potential from Matsubara sum:

$$\Phi_T = \frac{eB_0 T}{4\pi^2} \sum_{\ell=0}^{N} \int \mathrm{d}p_z \ln \left[1 - z_{B,\Omega} e^{-\beta\left(\frac{p_z^2}{2m_B} - \ell\Omega\right)} \right] \tag{12}$$

where $z_{B,\Omega} \equiv e^{\beta(\mu - \mu_\Omega - m_B)}$ is fugacity and magnetic mass is defined by $m_B = \sqrt{m^2 + eB_0}$.

★ **NOTE:** Given that BEC is a low-temperature phenomenon, a nonrelativistic dispersion relation was employed in the derivation.

★ After some calculation and simplification, we finally obtain

$$\Phi_T = -\frac{eB_0 TN}{2\pi\lambda_B} \mathrm{Li}_{3/2}(z_{B,\Omega}), \qquad \lambda_B \equiv \sqrt{2\pi/m_B T} \tag{13}$$

Number Density and BEC

★ Thermal part of number density

$$n_{\text{th}} = \frac{eB_0 N}{2\pi\lambda_B} \mathrm{Li}_{1/2}(z_{B,\Omega}) \tag{14}$$

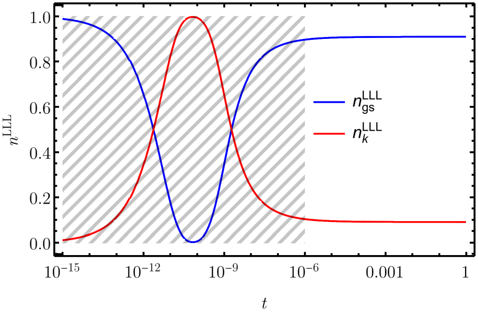
★ **Study BEC:** Weak scenario with magnetic field **[arXiv:2508.16799]**

$$n_{\text{th}} = n_{\text{gs}} + n_k \tag{15}$$

Assuming in ground state $|p_z| \leq |p_0|$

$$n_{\text{gs}} = \frac{eB_0 N}{2\pi\lambda_B} \sum_{j=1}^{\infty} \frac{z_{B,\Omega}^j}{\sqrt{j}} \mathrm{Erf}\left(\frac{p_0}{\sqrt{2m_B T}}\sqrt{j}\right) \tag{16}$$

Rotation and Magnetic Field Effects on a BEC



★ For this case, BEC can occur in TWO-STEPS! **[arXiv:2508.16799]**

★ For $B = 10^{15}\,G$, $\rho = 10^{-7} m_\pi^3$, $\Omega = 10^3\mathrm{Hz}$, and $N = \mathcal{O}(10^5)$.

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Thank You!

Back-up Slides

Rigid Rotation Metric

★ Uniform rotation around z-axis

★ In rotating frame

$$x'^{\mu} = (t, r\cos(\theta - \Omega t), r\sin(\theta - \Omega t), z) \quad (17)$$

★ The metric is given by

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (18)$$

where $r = \sqrt{x^2 + y^2}$.

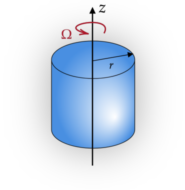

Figure: A rotating cylinder