An overview on phenomenological aspects of QCD

مروری بر جنبه های پدیده شناسی نظریه کیوسی‌دی

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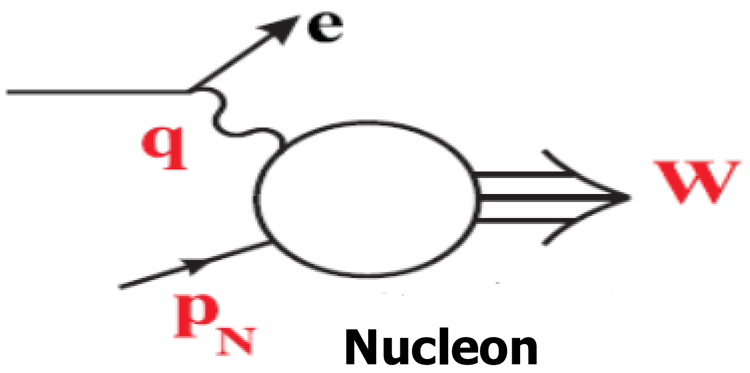
Deep inelastic scattering, a brief review



e-nucleon scatt.

Fig. 1

- High energy electron scattering is an ideal probe of the structure of a composite object .

- The photon is not on it`s mass shell, then q does not satisfy $q^2=0$.

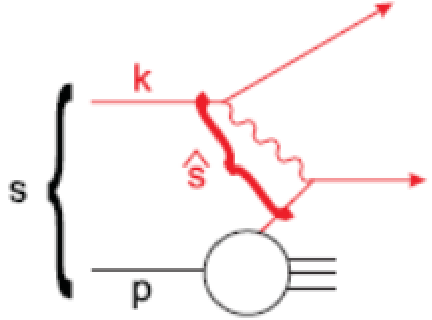
-The invariant mass W of the outgoing system :

$$W^2 = (p_N + q)^2 = M_N^2 + 2\,p_N.q + q^2 \qquad (1)$$

M_N and p_N are the mass and 4-momentum of the nucleus.

- Recall that by "deep" we mean $Q^2 \gg M^2$ and by "inelastic" we mean $W^2 = (p + q)^2 \gg M^2$.

The Quark Parton Model



- The basic idea of the QPM is that in the DIS process, ep → eX, the virtual proton interacts with one of the quark constituents of the proton.

- The photon "sees" the proton made up of the three quarks (called valence quarks) and an arbitrary number of $q\bar{q}$ pairs (made up of sea quarks). The sea quarks originate from gluons, via $g \rightarrow q\bar{q}$, themselves radiated from quarks.

Scales

Scaling violation

The Quark Parton Model

- the ep interaction may be written as an incoherent sum (of probabilities) of scattering from single free quarks :

$$\frac{d\sigma}{dxdQ^2} = \sum_q \int_0^1 d\xi \, f_q(\xi) \frac{d\tilde{\sigma}_{eq}}{dxdQ^2} \qquad (2)$$

- Where $f_q(\xi)$ is the probability of finding the quark q in the proton carrying a fraction ξ of its momentum.

The Quark Parton Model

- We have noted that the proton is made of valence quarks (uud) and sea quarks in $q\bar{q}$ pairs. When probed at a scale Q, all quark flavors with $m_q \leq Q$ are active.

- Usually the flavor is used as a shortened notation for a parton distribution. So, for example:

$$f_u(\mathrm{x}) \equiv u(\mathrm{x}) = \mathrm{u}_v(\mathrm{x}) + \mathrm{u}_{sea}(\mathrm{x}) \qquad (3)$$

$$f_{\bar{u}}(\mathrm{x}) \equiv \bar{u}(\mathrm{x}) = \mathrm{u}_{sea}(\mathrm{x})$$

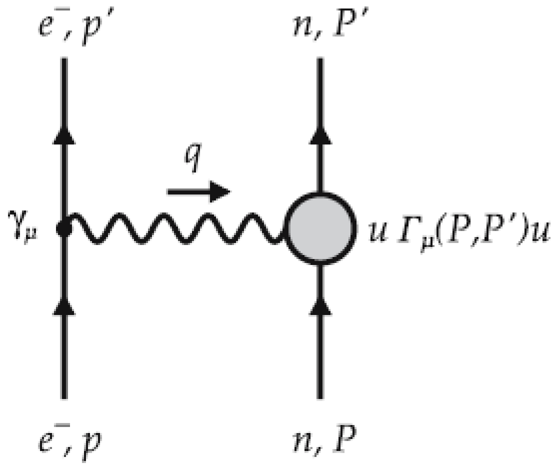
- We therefore have flavor sum rules :

$$\int_0^1 (\mathrm{u} - \bar{\mathrm{u}})\, dx = \int_0^1 u_V\, dx = 2 \qquad \int_0^1 (d - \bar{d})\, dx = \int_0^1 d_V\, dx = 1 \qquad (4)$$

The DIS observables: the structure functions

- The cross-section is of the form :

$$\frac{d\sigma}{dxdQ^2} = \frac{1}{x\,s}\frac{2\pi y\alpha^2}{Q^4}(\mathrm{L}_{\mu\nu}\mathrm{W}^{\mu\nu}) \qquad (3)$$



where $y = \frac{p.q}{p.k}$ and $s = (\mathrm{p}+k)^2 \simeq \frac{Q^4}{x\,y}$

- $L^{\mu\nu}$ is the tensor from the leptonic vertex known in terms of k and k′, and $W_{\mu\nu}$ is the unknown tensor describing the hadronic vertex.

The general form for $W_{\mu\nu}$ is

$$W_{\mu\nu} = (\mathrm{g}_{\mu\nu} + \frac{q_\mu q_\upsilon}{q^2})\mathrm{F}_1(\mathrm{x},Q^2) + \frac{P_\mu P_\upsilon}{p.q}F_2(\mathrm{x},Q^2) - \mathrm{i}\,\varepsilon_{\mu\nu\alpha\beta}\frac{q^\alpha q^\beta}{2\,p.q}F_3(\mathrm{x},Q^2)$$

Comparison (2) and (3) leads for instance, the F_2 structure function as it follows:

$$F_2^{ep} = x(\frac{4}{9}\mathrm{u} + \frac{1}{9}d + \frac{1}{9}s + \ldots + \frac{4}{9}\bar{\mathrm{u}} + \frac{1}{9}\bar{d} + \frac{1}{9}\bar{s} + \ldots)$$

The DIS observables: the structure functions

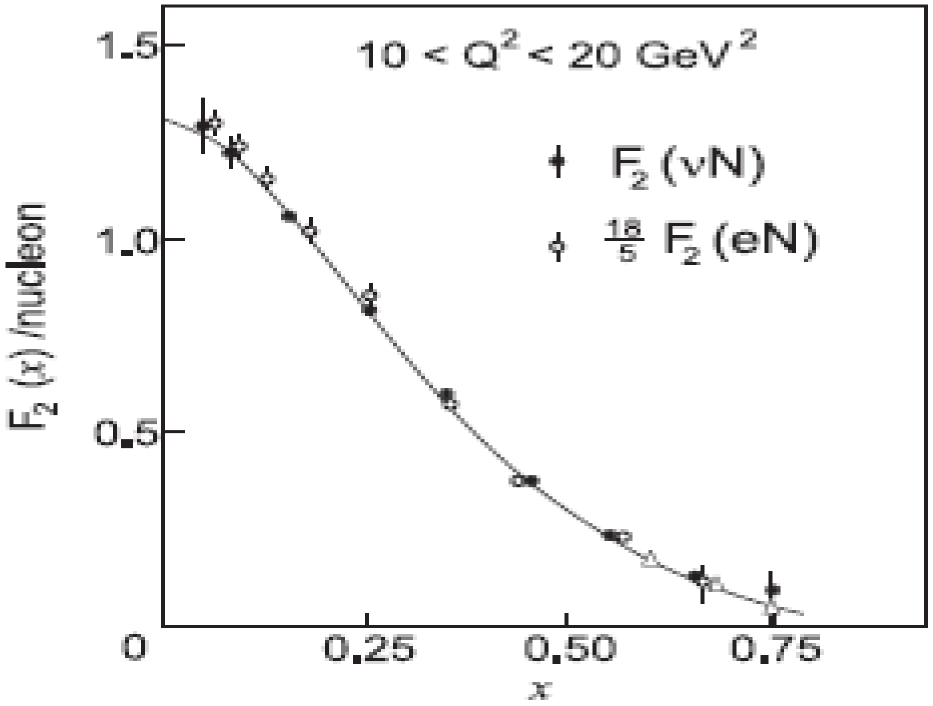


Fig.2

- An experimental comparison of DIS data in the early 1970s is shown in Fig. 2.

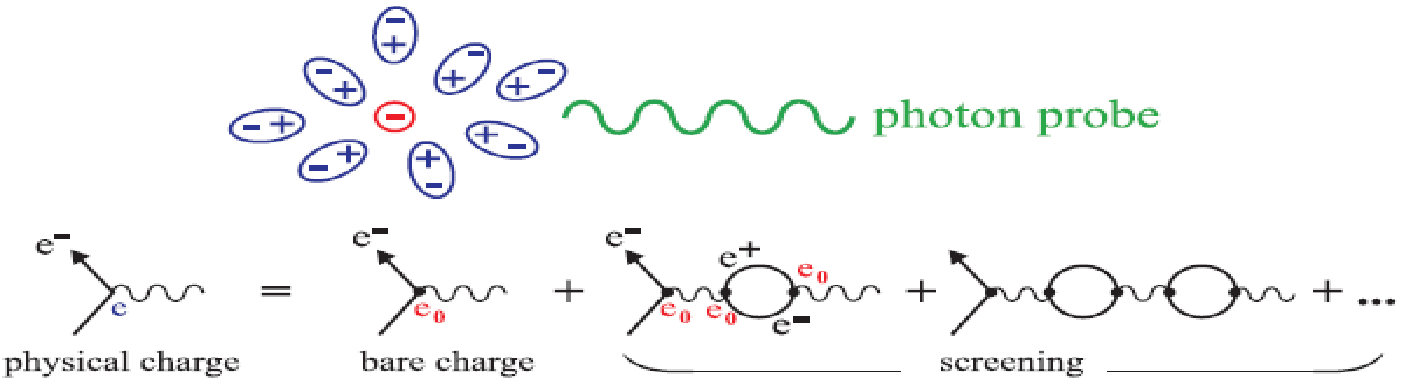
- The good agreement with the QPM relations is evident, but the area under the curve :

$$\int_0^1 F_2(e\mathrm{N})\,\mathrm{dx} = \int_0^1 \sum_{q,\bar{q}} x\, q(\mathrm{x}) \simeq 0.5$$

shows that only 50% of the proton's momentum is carried by quarks. It provided the first (indirect) evidence for the existence of the *gluonic* component of the proton.

The running QCD coupling

- First we discuss the QED coupling.



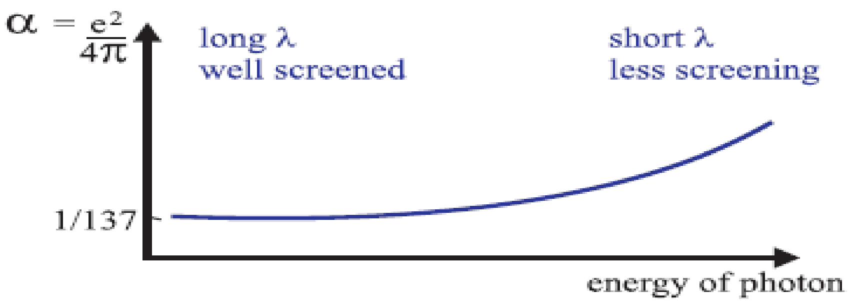
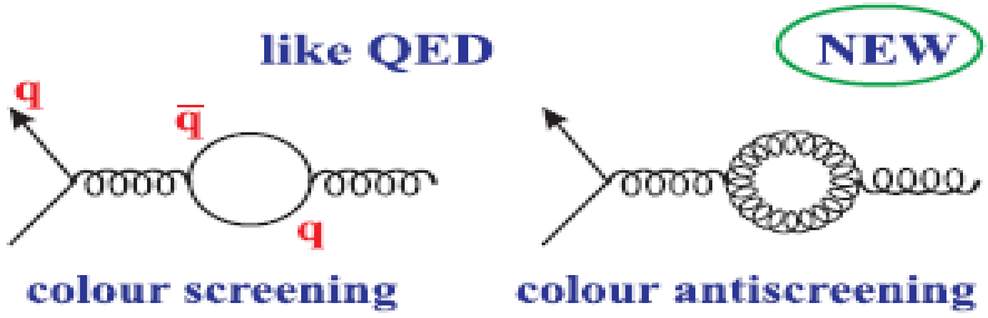


Fig. 3

- Vacuum polarisation effects (i.e. polarised e^+e^--pairs) screen the bare electron charge. The screening is least at short photon wavelengths, which causes the QED coupling, $\alpha = e^2/4\pi$, to increase with the energy of the photon.

The running QCD coupling

- Turning now to QCD we have a new vertex to consider, the triple-gluon vertex, which arises since the gluons themselves carry colour charge. This changes everything.



$$b_0 = \ -n_f/6\pi \quad + \ 33/12\pi$$

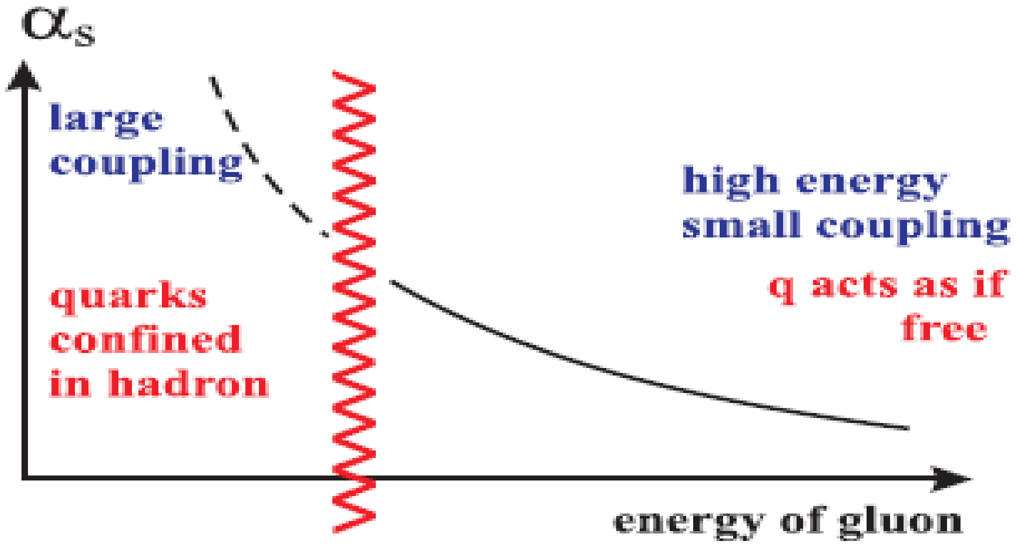


Fig. 4

Further discussion, DGLAP evolution equations

- Our $O(\alpha_s)$ would be completed if in addition to the $\gamma q \rightarrow gq$ subprocesse, at $O(\alpha_s)$, we need to include the $\gamma q \rightarrow q\bar{q}$ processes. Then the evolution equation (known as DGLAP equation) for the quark density $q \equiv f_q$ becomes :

$$\frac{\partial q(x,Q^2)}{\partial \log(Q^2)} = \frac{\alpha_s}{2\pi}(P_{qq} \otimes q + P_{qg} \otimes g) \qquad (6)$$

- Where $g \equiv f_g$ is the gluon density, and $P_{qq} \equiv P$ is the $q \rightarrow q(g)$ splitting function .

- Clearly we must also consider the evolution of the gluon density :

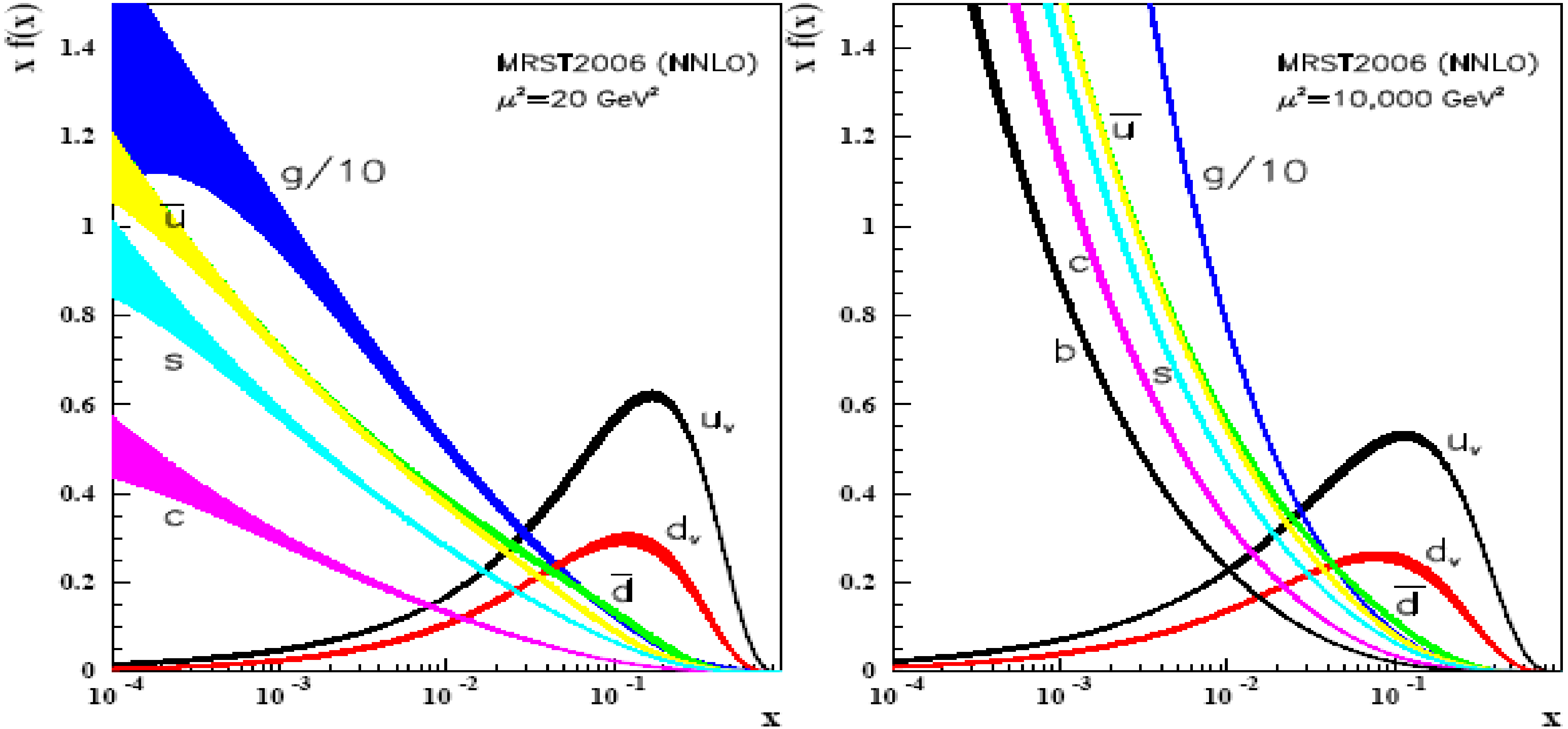
$$\frac{\partial g(x,Q^2)}{\partial \log(Q^2)} = \frac{\alpha_s}{2\pi}(\sum_i P_{gq} \otimes (q_i + \bar{q}_i) + P_{gg} \otimes g) \qquad (7)$$

The sum is over the i quark flavors, where P_{gq} and P_{gg} are $q \rightarrow gq$ and $g \rightarrow gg$ splitting functions.

Global parton analysis

- An example of the resulting parton distributions is shown in Fig. 3.

Fig. 5



Renormalization group equation

- The most crucial feature of QCD, as seen in DGLAP eqtaions, is the dependence of the QCD coupling, $\alpha_s \equiv g_2/4\pi$, on Q^2.

A scale enters when we use perturbation theory to calculate the observable :

$$R = \sum_{n=0}^{\infty} r_n a^{n+1}$$

- Since we encounter (loop) Feynman diagrams which diverge logarithmically. We need to renormalise (reparameterise) the theory, which introduces a renormalization scale μ. As a consequence we find that the dimensionless observable R has not longer scales, but has logarithmic scaling violations, that is, it has the functional dependence $R(\log(Q^2/\mu^2), \alpha_s(\mu^2))$.

R cannot depend on the choice of renormalisation scale, so we have a renormalisation group equation (RGE) :

$$\frac{dR}{d \log \mu^2} = \left(\frac{\partial}{\partial \log \mu^2} + \frac{\partial \alpha_s}{\partial \log \mu^2} \frac{\partial}{\partial \alpha_s}\right) R = 0$$

The renormalization group summation(RGS) approach to avoid the scale and scheme dependence

$$R=\sum_{n=0}^{\infty} r_n a^{n+1},\ \ r_n=\sum_{m=0}^{n} T_{n,m}L^m \ \ where\ \ L=\ln(\mu/Q) \Rightarrow R=\sum_{n=0}^{\infty}\sum_{m=0}^{n} T_{n,m}L^m a^{n+1}$$

A new grouping: $A_n=\sum_{m=0}^{\infty} T_{n+m,n}a^{n+m+1} \Rightarrow R=\sum_{n=0}^{\infty} A_n(a)L^n$

This makes a possibility to sum the contribution to R, considering the RGE.

$$RGE \Rightarrow \sum_{n=0}^{\infty}(bnA_n(a)L^{n-1}+\beta(a)A_n'(a)L^n)=0 \ \Rightarrow A_n(a)=-\frac{\beta(a)}{nb}\frac{d}{da}A_{n-1}(a) \quad (1)$$

Where $\beta(a)=\frac{\partial a}{\partial \ln \mu^2}=-ba^2(1+ca+c_2a^2+c_3a^3+\ldots)$ (2) Known as QCD $\beta-function$

(1) and (2) $\Rightarrow A_n(a(\ln\frac{\mu}{\Lambda}))=-\frac{1}{n}\frac{d}{d\ln(\frac{\mu}{\Lambda})}A_{n-1}(a(\ln\frac{\mu}{\Lambda}))$

Substituting this into the expression for R ,will lead us to

$$R=\sum_{n=0}^{\infty}A_n(a)L^n,\ A_n(a(\ln\frac{\mu}{\Lambda}))=-\frac{1}{n}\frac{d}{d\ln(\frac{\mu}{\Lambda})}A_{n-1}(a(\ln\frac{\mu}{\Lambda}))\Rightarrow R=\sum_{n=0}^{\infty}\frac{1}{n!}(-\frac{L}{b})^n\frac{d^n}{d\eta^n}A_0(a(\eta))=A_0(a(\eta-\frac{L}{b}))$$

$$L=\ln(\mu/Q)\,,\ \eta=\ln\frac{\mu}{\Lambda}\,,\ R=A_0(a(\eta-\frac{L}{b}))\Rightarrow R=A_0(a(\ln\frac{Q}{\Lambda}))$$

This shows that all dependence of R on the μ scale has been cancelled.

This is a pleasant result.

By doing a full resummation on the QCD perturbative series, unphysical μ scale has been removed.

How to remove the scheme dependence in RGS approach

How the perturbative series of QCD observable could be rearranged to yield us a result which would be scheme independent?

The coefficients of QCD β-function, c_i , are scheme dependent and could be used to parametrized the renormalization scheme (RS).

$$\frac{dR}{dc_i}=\left(\frac{\partial}{\partial c_i}+\frac{\partial a}{\partial c_i}\frac{\partial}{\partial a}\right)R=0\,,\quad \frac{\partial a}{\partial c_i}=\beta_i(a)=\frac{1}{i-1}a^{i+1}\left[1-\frac{(i-2)}{i}ca+\left(\frac{(i-1)(i-2)}{i(i+1)}c^2-\frac{(i-3)}{(i+1)}c_2\right)a^2+\ldots\right]\,,\quad R=\sum_{n=0}^{\infty}T_n a^{n+1}$$

(According to new grouping)

$$\Rightarrow \sum_{n=0}^{\infty}\left(a^{n+1}\frac{\partial T_n}{\partial c_i}+(n+1)\beta_i(a)T_n a^n\right)=0 \quad (3) \Rightarrow$$

$$\frac{\partial T_0}{\partial c_i}=0,\quad \frac{\partial T_1}{\partial c_i}=0,\quad \frac{\partial T_2}{\partial c_2}+1=0\,,$$

$$\frac{\partial T_3}{\partial c_2}+2\tau_1=0,\quad \frac{\partial T_3}{\partial c_3}+\frac{1}{2}=0\,,$$

$$\frac{\partial T_4}{\partial c_2}+\frac{1}{3}c_2+3T_2=0,\quad \frac{\partial T_4}{\partial c_3}+T_1-\frac{c}{6}=0,\quad \frac{\partial T_4}{\partial c_4}+\frac{1}{3}=0\,,\ \ldots$$

$$\Rightarrow T_0=\tau_0=1,\quad T_1=\tau_1=Const,\quad T_2=-c_2+\tau_2,\quad T_3=-2c_2\tau_1-\frac{1}{2}c_3+\tau_3,\quad T_4=-\frac{1}{3}c_4-c_3(\tau_1-\frac{c}{6})+\frac{4}{3}c_2^2-3c_2\tau_2+\tau_4,\ldots$$

$$(\tau_i \ are \ constant \ of \ integration)$$

Substituting the results for T_n in R, the final result would be definitely independent of any RS at truncated orders since the upcoming results for T_n coefficients are based on Eq.(3) which guarantees this feature.

This RS independent, provide us a freedom to choose any scheme which make the computations simplify. On this base 't Hooft scheme in which $c_i=0\ (i\geq 2)$ is chosen and final result would be:

$$R=\sum_{n=0}^{\infty}\tau_n a^{n+1}(\ln\frac{Q}{\Lambda})=a(\ln\frac{Q}{\Lambda})+\tau_1 a^2(\ln\frac{Q}{\Lambda})+\tau_2 a^3(\ln\frac{Q}{\Lambda})+\ldots\ .$$

As we expected, the final result for R observable in RGS approach is independent of any renormalization scale and scheme.

Scale and scheme independent in Complete Renormalization Group improvement approach

The requirements to establish the Complete Renormalization Group improvement (CORGI) approach includes:1- Self consistency condition (SCC) for perturbative series 2- Trade the renormalization scale, r_1 , with first perturbatuve coefficient, μ , using SCC. 3-Extending the scheme parameters which in addition to c_i parameters, includes r_1 .

1- SCC: $R = R^{(i)} + O(a^{i+1})$, $R = R^{(i)'} + O(a^{i+1})$ $\Rightarrow \dfrac{\partial R^{(i)}}{\partial (RS)} = O(a^{i+1})$ When some calculations are done in two different schemes at N^iLO order, the results should be differed by the order a^{i+1} .

2- $\tau = Ln(\mu/\Lambda_{\overline{MS}})$, SCC $\Rightarrow \tau - r_1 = \rho_0(Q)$ r_1 and τ could be traded.

3- $SCC \Rightarrow$

$$r_2(r_1,c_2) = r_1^2 + cr_1 + X_2 - c_2 ,$$

$$r_3(r_1,c_2,c_3) = r_1^3 + \frac{5}{2}cr_1^2 + (3X_2 - 2c_2)r_1 + X_3 - \frac{1}{2}c_3 ,$$

$\vdots \quad \vdots$

General structure $\Rightarrow r_n(r_1,c_2,\ldots,c_n) = \hat{r}_n(r_1,c_2,\ldots,c_{n-1}) + X_n - c_n/(n-1)$. X_n are constants of integrations and RS invariants.

Substituting the result for $r_2(r_1,c_2),\ r_3(r_1,c_2,c_3),\ \ldots$ in $R=\sum_{n=0}^{\infty} r_n a^{n+1}$ expansion, gives:

$$R(Q)=a+r_1a^2+(r_1^2+cr_1+X_2-c_2)a^3+(r_1^3+\frac{5}{2}cr_1^2+(3X_2-2c_2)r_1+X_3-\frac{1}{2}c_3)a^4+\ldots \quad (4)$$

where a has the dependence as $a\equiv a(r_1,c_2,c_3,\ldots)$.

Main idea of CORGI approach: The R(Q) observable is scale and scheme independence and hence the NLO, N^2LO and … contributions of right hand side of Eq.(4) would also be scale and scheme independence.

NLO contribution $\Rightarrow$ $a_0(\mathrm{Q})\equiv a+r_1a^2+(r_1^2+cr_1-c_2)a^3+(r_1^3+\frac{5}{2}cr_1^2-2c_2r_1-\frac{1}{2}c_3)a^4+\ldots$ (5) (at NLO contribution, X_2 and X_3 are unknown)

The invariant subset of Eq.(5) can be computed in the favorable scheme like the \`t Hooft scheme in which $c_2=c_3=\ldots=0$ while it is considered as well that $r_1=0$ $\Rightarrow a_0=a(r_1=0,c_2=0,c_3=0,\ldots,c_n=0)$.

N^2LO contribution $\Rightarrow$ $X_2a_0^3=X_2a^3+3X_2r_1a^4+\ldots$ (here the same assumption is used while at N^2LO contribution, X_2 is known)

This procedure can be extended to any higher order: $\Rightarrow$ $R(Q)=a_0+X_2a_0^3+X_3a_0^4+\ldots+X_na_0^{n+1}+\ldots$

Here the sum is scale and RS independent at any order of approximation.

Application of RGS and CORGI approaches for thermal free energy density in QCD

Thermal free energy density in QCD describes the behavior of the quark-gluon plasma (QGP) system at finite temperature. It can be used to determine the equation of state in QCD, which expresses the relationship between pressure, energy density, and temperature.

For computational purpose the thermal free energy density can be divided ro three parts

$$F(T)/F_0 = 1 + \sum_{n=0}^{\infty} r_n a^{(n+1)} + \sum_{n=0}^{\infty} s_n a^{(n+\frac{3}{2})} + \sum_{n=0}^{\infty} t_n a^{(n+2)} \ln a$$

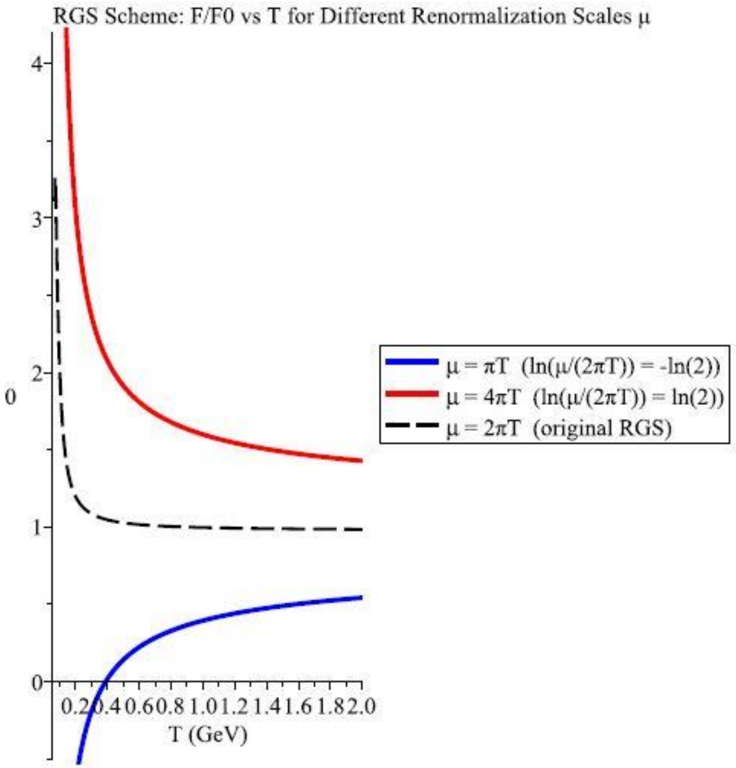
RGS approach $\Rightarrow$ $F(T)/F_0 = 1 + \Omega_0 a + \Gamma_0 a^{\frac{3}{2}} + \Upsilon_0 a^2 \ln a + \Omega_1 a^2 + \Gamma_1 a^{\frac{5}{2}} + \Upsilon_1 a^3 \ln a + \Omega_2 a^3 + \cdots$

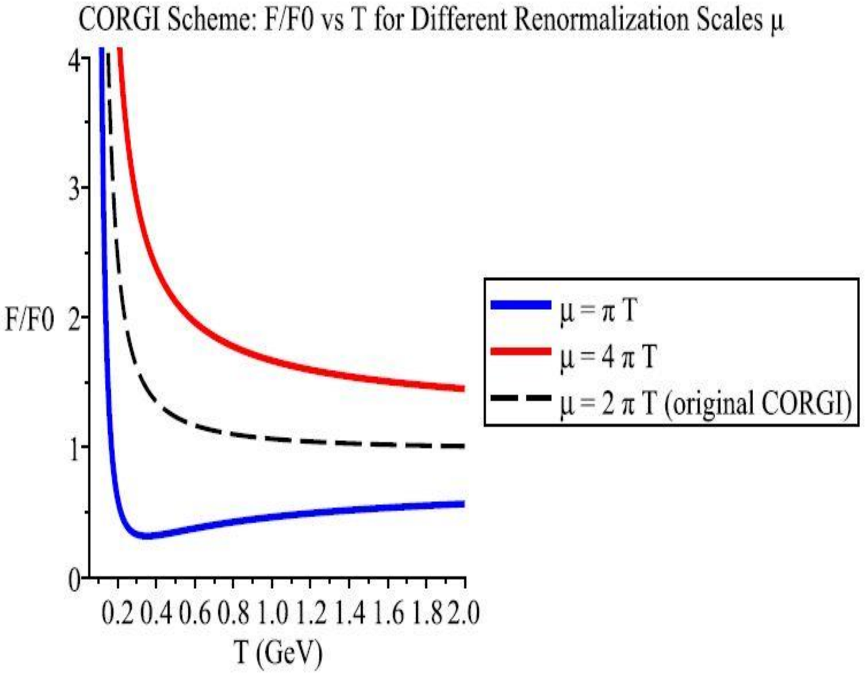
Here $\Omega_0, \Gamma_0, \Upsilon_0, \Omega_1, \Gamma_1$ *and* Υ_1 are constant and RS invariants and $a = a(\frac{2\pi T}{\Lambda})$.

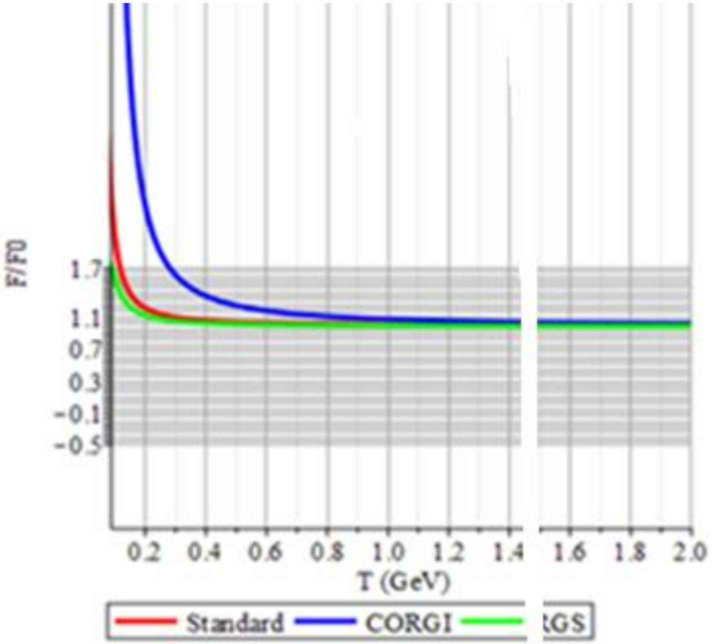
COTGI approach $\Rightarrow$ $F(\mathrm{T})/F_0 = 1 + a_0 + X_2 a_0^3 + X_3 a_0^4 + a_0^{3/2} + \Gamma_2 a_0^{7/2} + \Gamma_3 a_0^{9/2} + a_0^2 \ln a + \Omega_2 a_0^4 \ln a + \Omega_3 a_0^5 \ln a + \cdots$

Here $X_2, X_3, \Gamma_2, \Gamma_3, \Omega_2$ *and* Ω_3 are constant and RS integrations and $a_0 = a_0(\frac{2\pi T}{\Lambda})$ at two order of approximation

Results


RGS Scheme: F/F0 vs T for Different Renormalization Scales μ
μ = πT (ln(μ/(2πT)) = -ln(2))
μ = 4πT (ln(μ/(2πT)) = ln(2))
μ = 2πT (original RGS)
T (GeV)


CORGI Scheme: F/F0 vs T for Different Renormalization Scales μ
F/F0
μ = π T
μ = 4 π T
μ = 2 π T (original CORGI)
T (GeV)


F/F0
T (GeV)
Standard
CORGI

Conclusion

1- The important of QPM as a tool to consider DIS proses was reviewed.

2- The partonic DGLAP evolution equation to achieve to nucleon structure function was considered.

3- The role of running coupling constant was paid attend to make the calculations at higher order approximations.

4- Extending the calculations to higher order approximations make the coupling constant and perturbative coefficients to depend on renormalization scale and scheme.

5- Two different RGS and CORGI approaches have been used to avoid from unphysical dependences.

6- The result of CORGI approach for thermal free energy density was drastically far from the result of RGS approach and conventional pQCD which indicates the priority of CORGI approach with respect to RGS.

Thanks for your attention