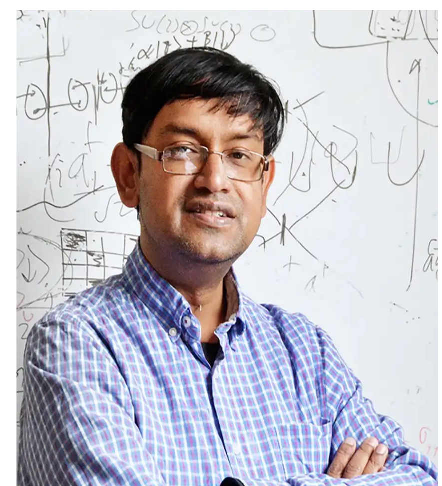
电子科技大学
University of Electronic Science and Technology of China

ISFAHAN UNIVERSITY OF TECHNOLOGY
دانشگاه صنعتی اصفهان

Quantum-Enhanced Sensitivity through Many-Body Bloch Oscillations

Hassan Manshouri, Moslem Zarei, Mehdi Abdi, Sougato Bose, and Abolfazl Bayat

Quantum 9, 1793 (2025).



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OUTLINE

- Critical Quantum Metrology
- Stark Localization (Single particle)
- Time evolution of Stark Localization
- Scaling of the Quantum Fisher Information
- Stark Many-body Localization (MBL)
- Practical implementation

Stark Localization
Weak Field Sensing

PHYSICAL REVIEW LETTERS **131,** 010801 (2023)

Stark Localization as a Resource for Weak-Field Sensing with Super-Heisenberg Precision

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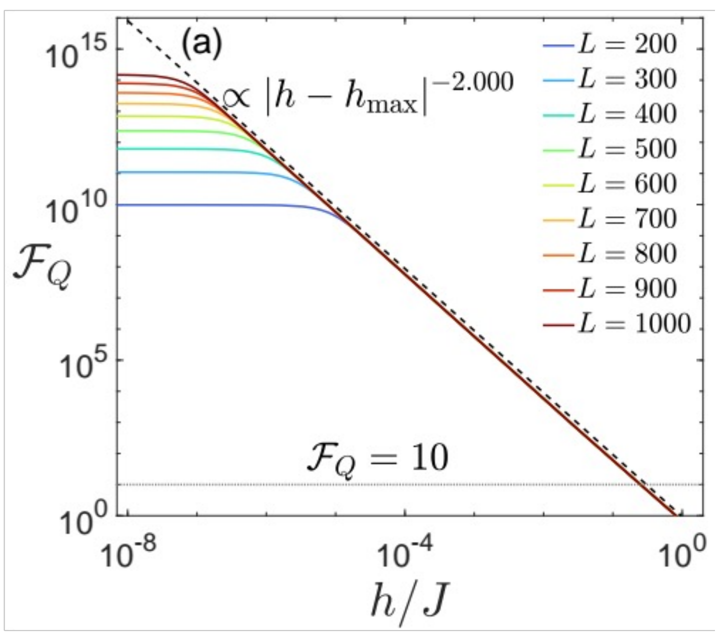
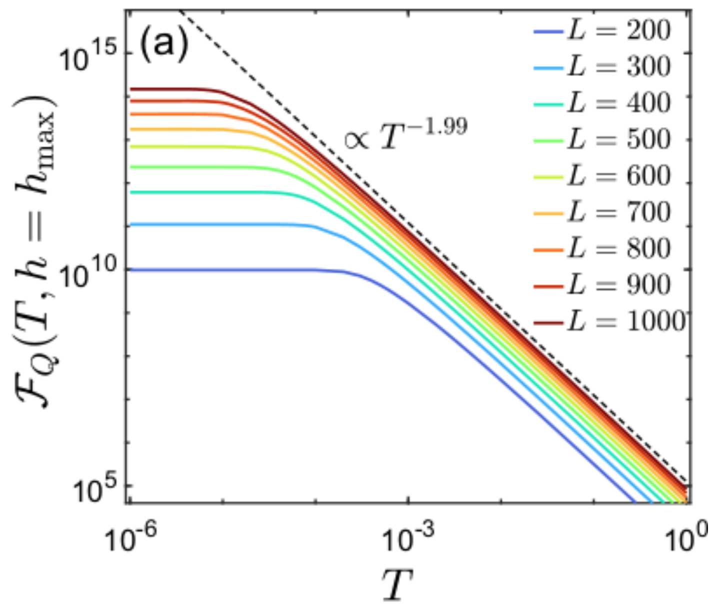




Quantum Fisher Information (QFI) and Cramer-Rao bound

In mathematical statistics, the Fisher information is a way of measuring the amount of information that an observable random variable X carries about an unknown parameter *h* of a distribution that models X.

$$\mathcal{F}_C(h) = \sum_x \frac{1}{p_x(h)} \left(\frac{dp_x}{dh} \right)^2$$

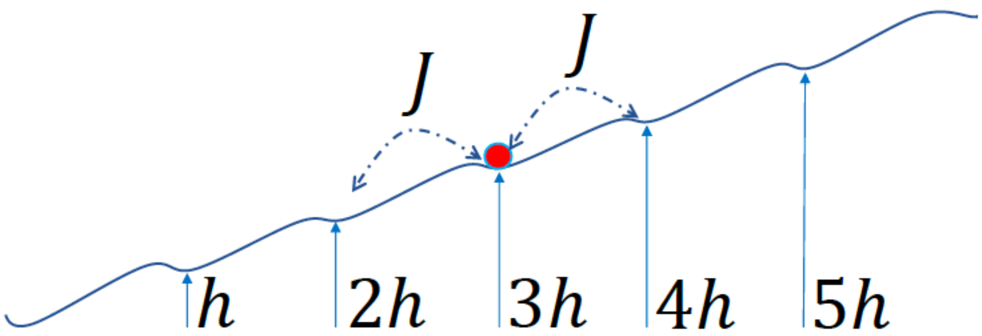
$$\mathcal{F}_Q(h) = 4[\langle \partial_h \Psi | \partial_h \Psi \rangle - | \langle \partial_h \Psi | \Psi \rangle |^2]$$

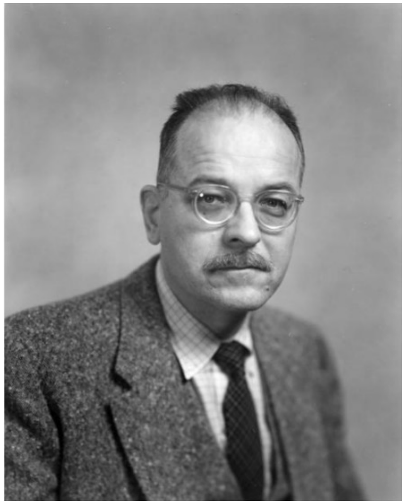
$$\delta h \geq \frac{1}{\sqrt{\mathcal{M} \mathcal{F}_C}} \geq \frac{1}{\sqrt{\mathcal{M} \mathcal{F}_Q}}$$

the saturation of the first inequality requires an optimal estimator, the saturation of the second inequality demands both optimized measurement and estimator.

Wanier-Stark localization

$$H(h) = J\sum_{i=1}^{L-1}(|i\rangle\langle i+1| + |i+1\rangle\langle i|) + h\sum_{i=1}^{L} i|i\rangle\langle i|$$





G. H. Wannier, Phys. Rev. 117, 432 (1960)

Wanier-Stark localization
Bloch oscillation

$$H(h) = J\sum_{i=1}^{L-1}(|i\rangle\langle i+1| + |i+1\rangle\langle i|) + h\sum_{i=1}^{L} i|i\rangle\langle i|$$

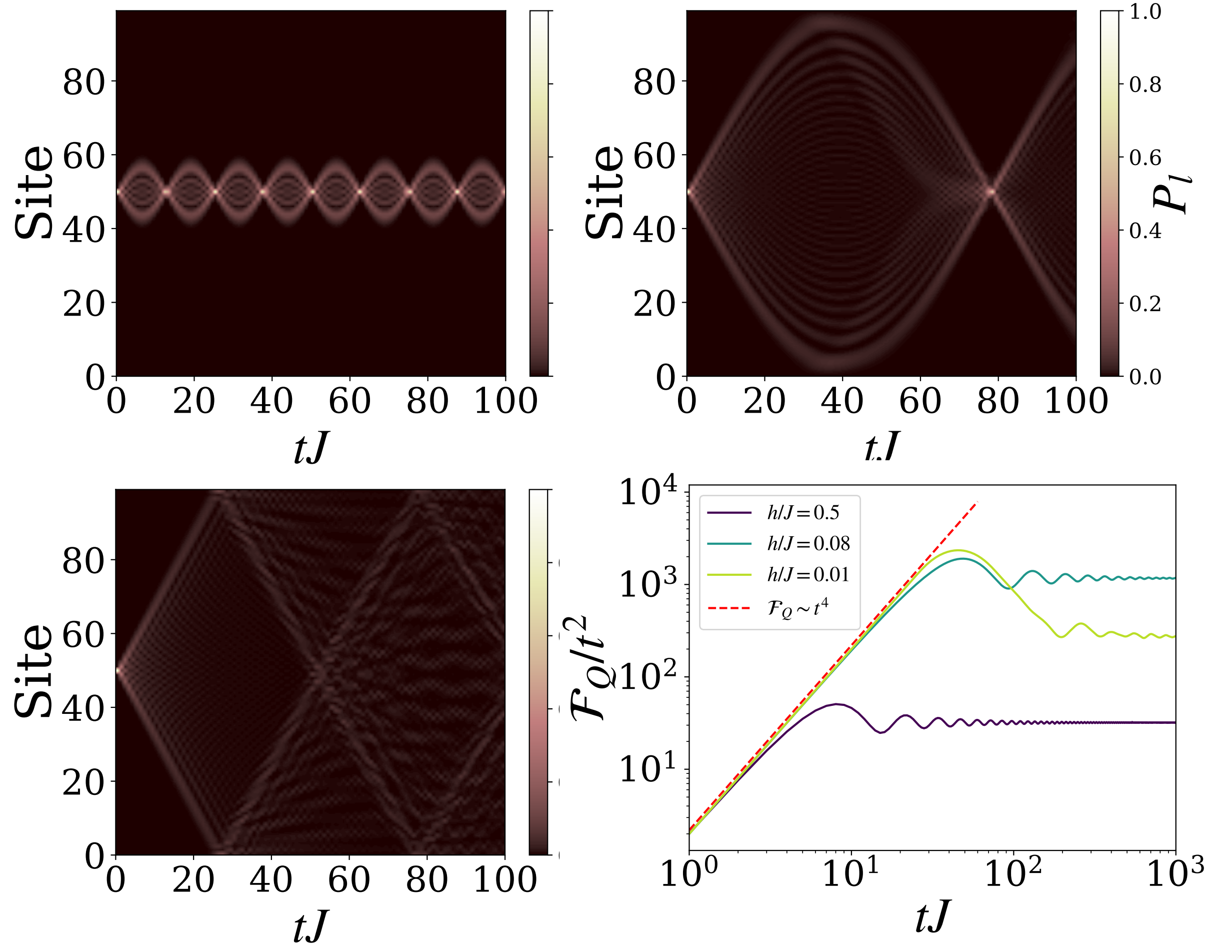
$$|\Psi(0)\rangle = \sum_l f_l\,|l\rangle$$

$$|\Psi(t)\rangle = \sum_{l,l'=1}^{L} \mathcal{K}_{l,l'}(t) f_{l'}\,|l\rangle$$

$$\mathcal{K}_{l,l'}(t) = \sum_{m=1}^{L} \langle l|E_m\rangle\langle E_m|l'\rangle\, e^{-iE_m t}$$

$$|E_m\rangle = \sum_{l=1}^{L} \mathcal{J}_{l-m}\left(\frac{2J}{h}\right)|l\rangle$$

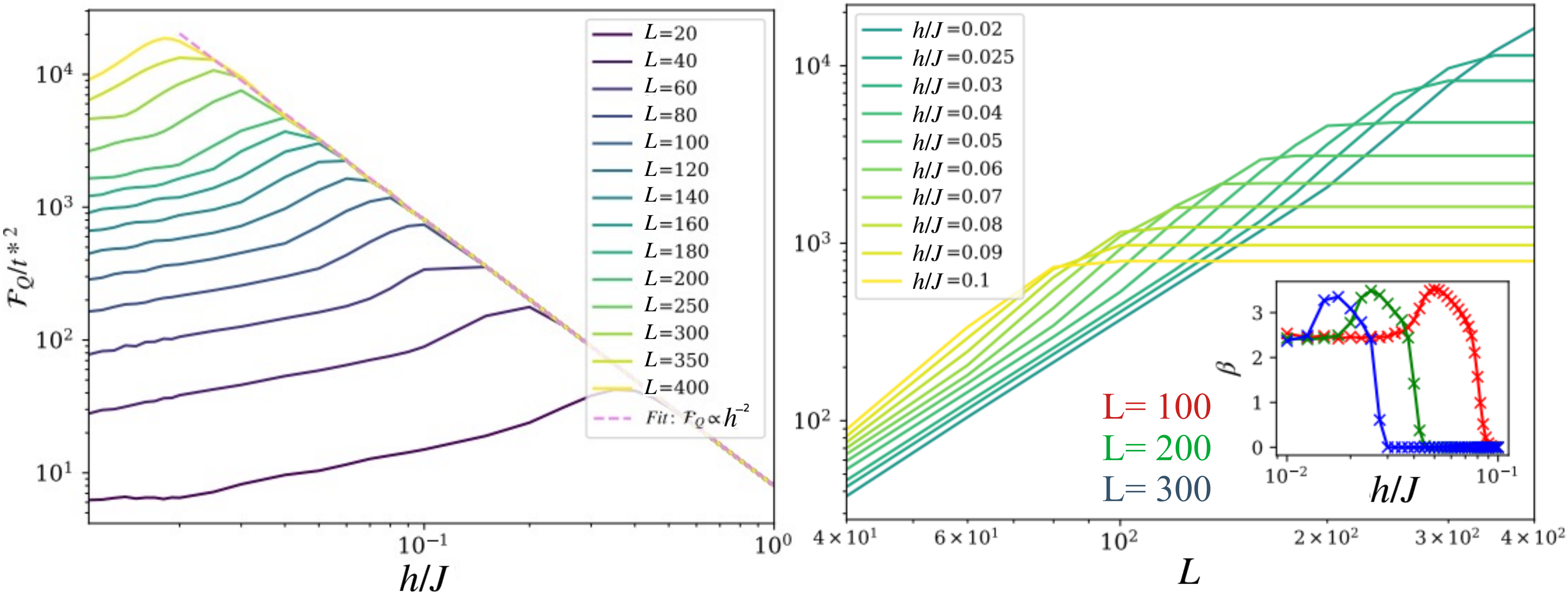
$$E_m = mh$$



Martin Holthaus & Daniel W. Hone, Philosophical Magazine B, 74:2, 105-137

Quantum Fisher Information (QFI)
Size scaling

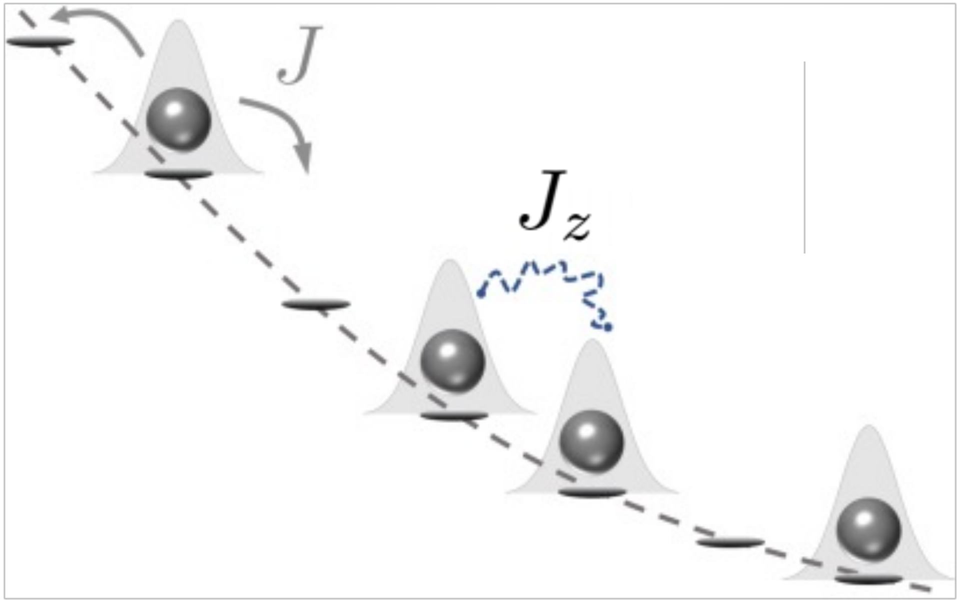
$$H(h) = J \sum_{i=1}^{L-1} (|i\rangle\langle i+1| + |i+1\rangle\langle i|) + h \sum_{i=1}^{L} i|i\rangle\langle i|$$



$$\mathcal{F}_Q \sim h^{-2} t^2 L^0, \quad \text{for } h > h_c$$
$$\mathcal{F}_Q \sim t^2 L^\beta, \quad \text{for } h \leq h_c$$

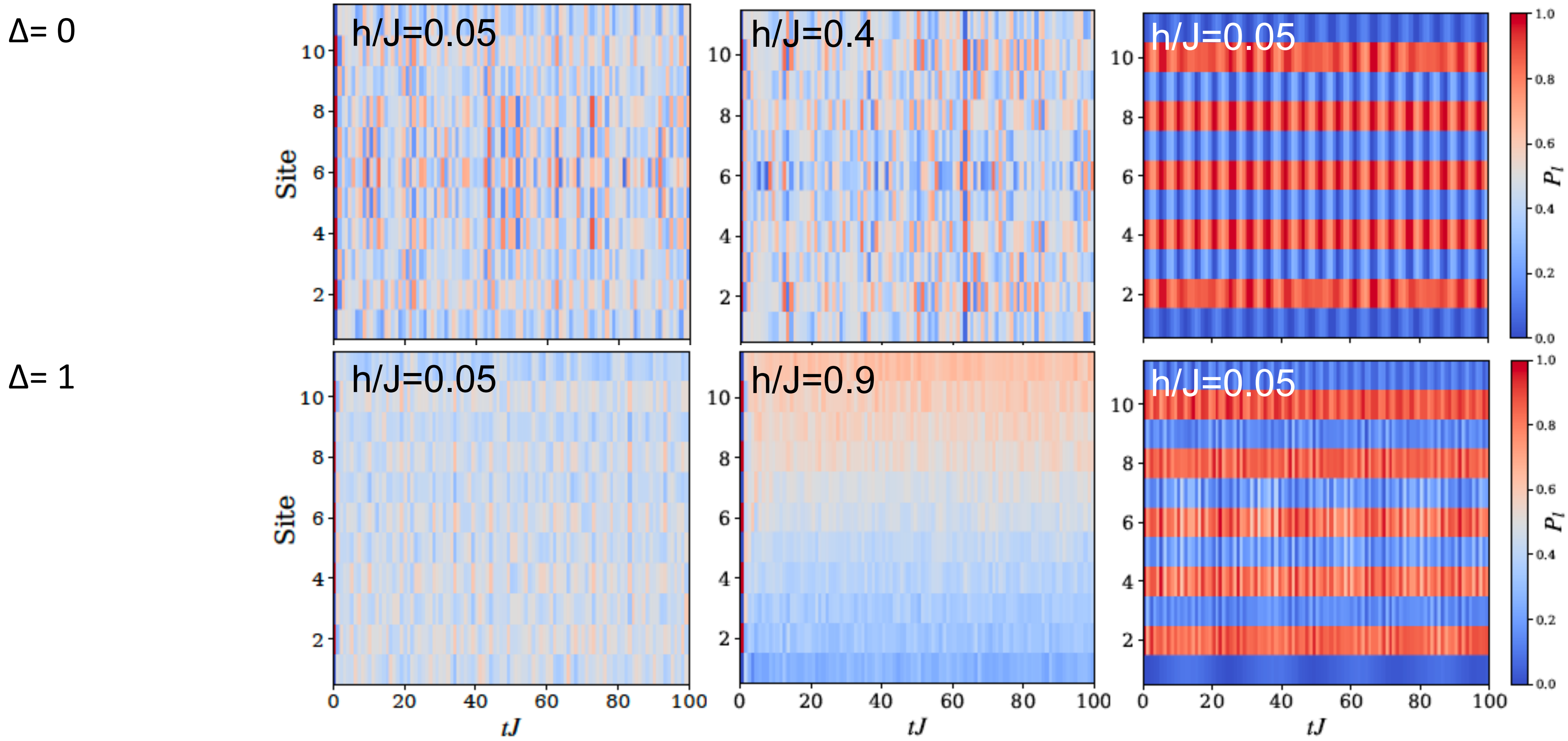
Stark many-body localization (Stark MBL)

$$H=-\frac{J}{2}\sum_{l=1}^{L-1}(\sigma_l^x\sigma_{l+1}^x+\sigma_l^y\sigma_{l+1}^y+\Delta\sigma_l^z\sigma_{l+1}^z)+h\sum_{l=1}^{L}l\sigma_l^+\sigma_l^-$$



Stark many-body localization
Many-Body Bloch oscillation

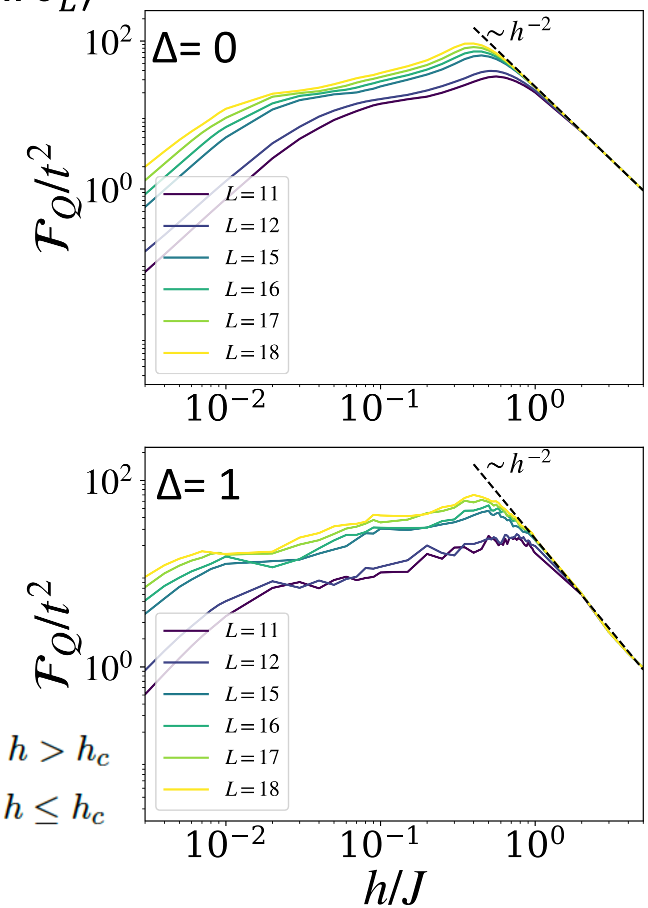
$$H = -\frac{J}{2}\sum_{l=1}^{L-1}(\sigma_l^x\sigma_{l+1}^x + \sigma_l^y\sigma_{l+1}^y + \Delta\sigma_l^z\sigma_{l+1}^z) + h\sum_{l=1}^{L} l\sigma_l^+\sigma_l^-$$



Stark many-body localization
QFI

$$H = -\frac{J}{2}\sum_{l=1}^{L-1}(\sigma_l^x\sigma_{l+1}^x + \sigma_l^y\sigma_{l+1}^y + \Delta\sigma_l^z\sigma_{l+1}^z) + h\sum_{l=1}^{L} l\sigma_l^+\sigma_l^-$$

$|0_0 \dots 0101010 \dots 0_L\rangle$



$|0_0 \dots 010 \dots 0_{17}\rangle$

$|0_0 \dots 01010 \dots 0_{17}\rangle$

$|0_0 \dots 0101010 \dots 0_{17}\rangle$

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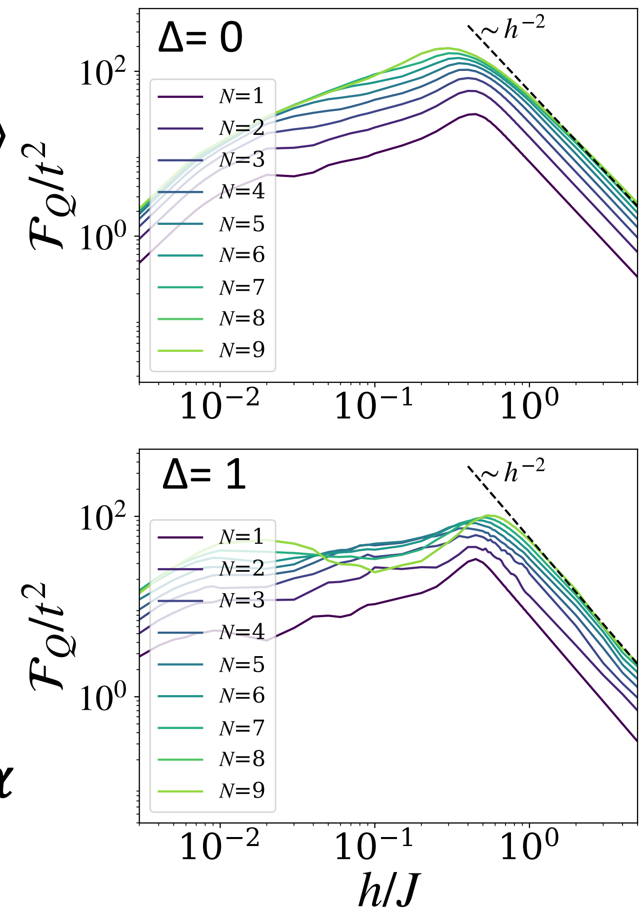


$\mathcal{F}_Q \sim h^{-2}t^2L^0, \quad \text{for } h > h_c$

$\mathcal{F}_Q \sim t^2L^\beta, \quad \text{for } h \le h_c$

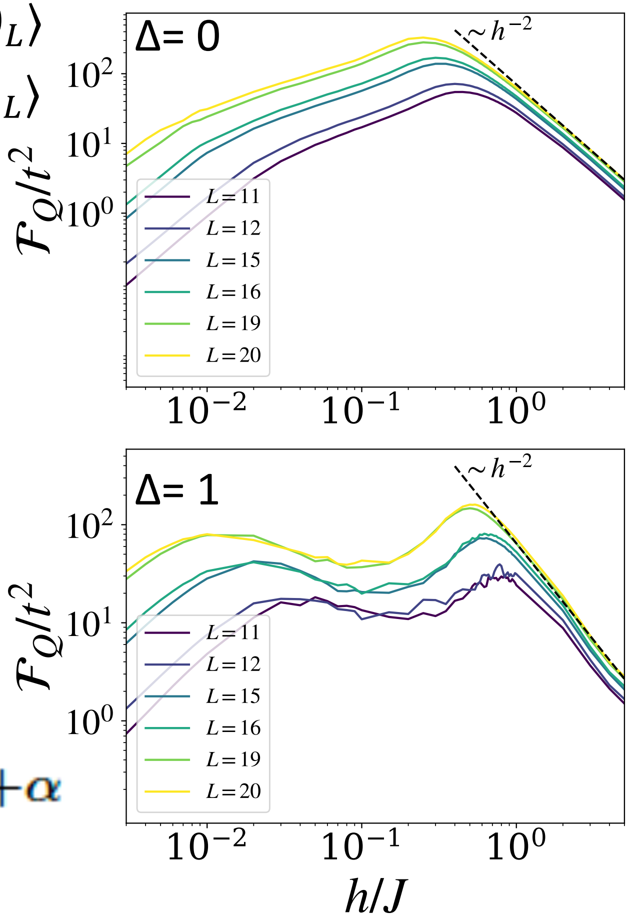
$$\mathcal{F}_Q \sim t^2N^\alpha$$

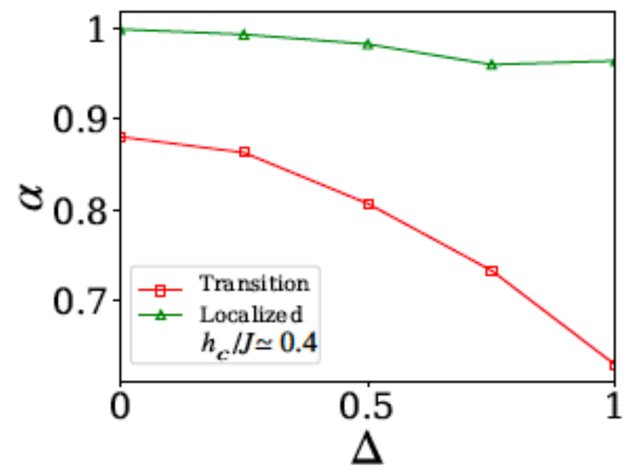
Stark many-body localization
QFI

$$H = -\frac{J}{2}\sum_{l=1}^{L-1}(\sigma_l^x\sigma_{l+1}^x + \sigma_l^y\sigma_{l+1}^y + \Delta\sigma_l^z\sigma_{l+1}^z) + h\sum_{l=1}^{L} l\sigma_l^+\sigma_l^-$$

$odd\ L\text{:}\quad |01 \ldots 10_L\rangle$

$even\ L\text{:}\ |01 \ldots 10_L\rangle$





$$\mathcal{F}_Q \sim t^2 L^{\beta+\alpha}$$

$$\mathcal{F}_Q \sim h^{-2}t^2L^0N^\alpha, \quad \text{for } h > h_c$$
$$\mathcal{F}_Q \sim t^2L^\beta N^\alpha, \quad \text{for } h \leq h_c$$

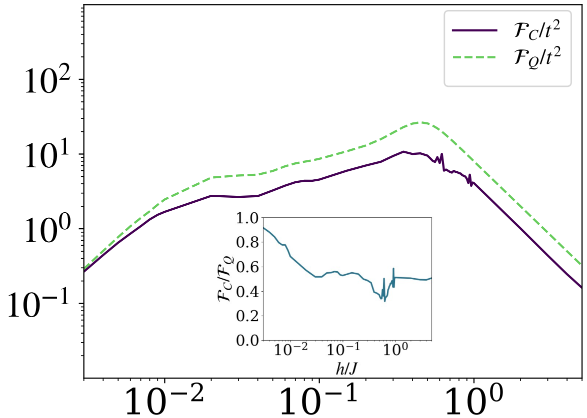
Stark localization
Practical implementation

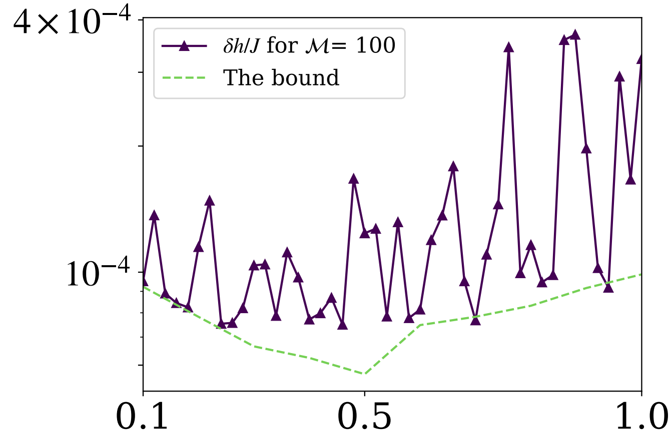
$p_{\mathbf{z}}(t) = |\langle \mathbf{z}|\Psi(t)\rangle|^2 \quad P(X|h) = \binom{\mathcal{M}}{n_0, n_1, \ldots,} \prod_{\mathbf{z}} p_{\mathbf{z}}^{n_{\mathbf{z}}} \quad h_{es} = \arg\max_{\{h\}} P(X|h)$

Classical Fisher information

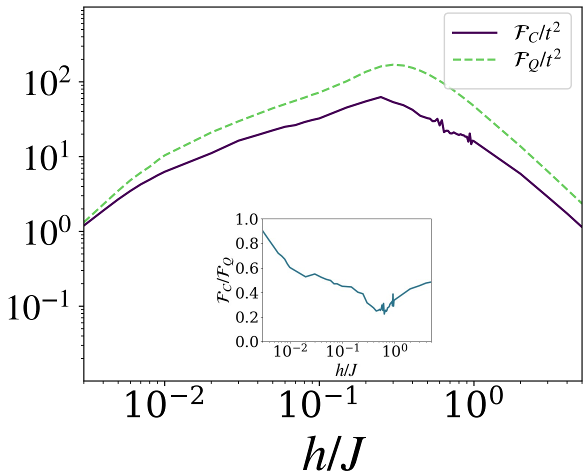
Maximum likelihood estimation

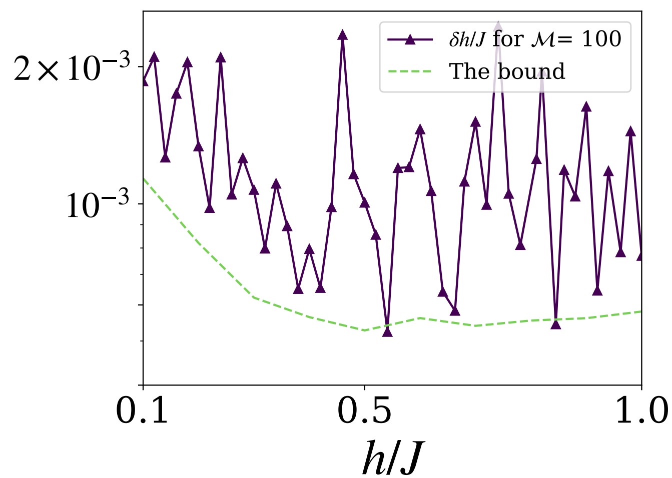
$|0 \ldots 010 \ldots 0\rangle$





$|01 \ldots 10_L\rangle$





Conclusion

- Quantum-enhance scaling of Stark Model
- Accessibility of Stark model in experiment
- Practical measurement
- More applicable enhancement
- Varies application in Scientific research

Thank you for your attention